

# Hybrid Beamforming for 5G Millimeter Wave Systems

**Jun Zhang**



# Collaborators



**Xianghao Yu**  
(FAU)



**Juei-Chin Shen**  
(MediaTek)



**Khaled B. Letaief**  
(HKUST)

- ❖ **Background and Motivation**
- ❖ **Preliminaries of Hybrid Beamforming**
- ❖ **Hybrid Beamforming Design**
  - **Improve Spectral Efficiency: Approaching the Fully Digital**
  - **Boost Computational Efficiency: Convex Relaxation**
  - **Fight for Hardware Efficiency: How Many Phase Shifters Are Needed?**
- ❖ **Conclusions**
- ❖ **Potential Research Directions**

# Background and Motivation

## ❖ Era of mobile data deluge

**7x**

Data growth by  
2021



**8.0 Billion**

Mobile devices/connections  
in 2016



**60%**

Video traffic in 2016

Cisco VNI, March 2017



# Background and Motivation

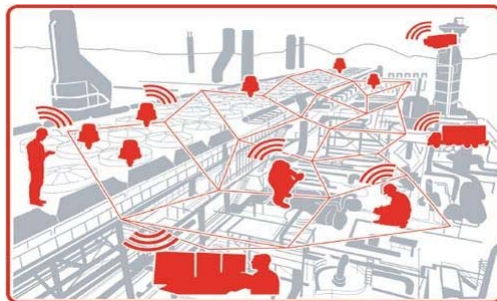
## ❖ Requirements of 5G systems



High data rate



Massive connections



Uniform coverage



Green communications



Security & privacy

# Background and Motivation

## ❖ The 1000x Capacity Challenge for 5G

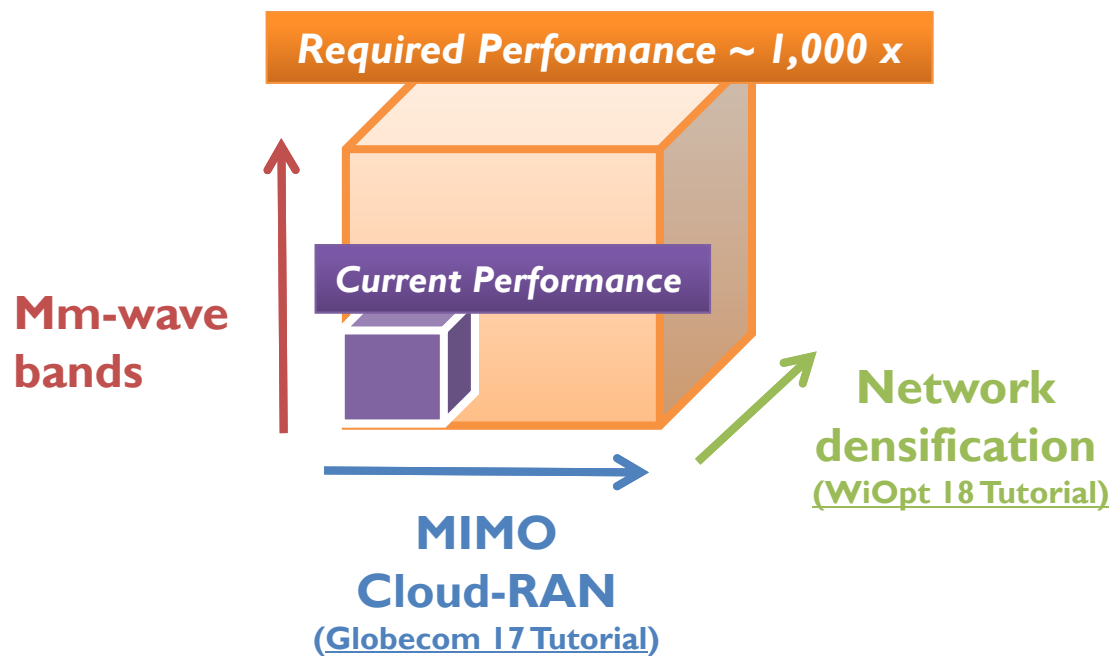


# Background and Motivation

## ❖ The 1000x Capacity Challenge for 5G

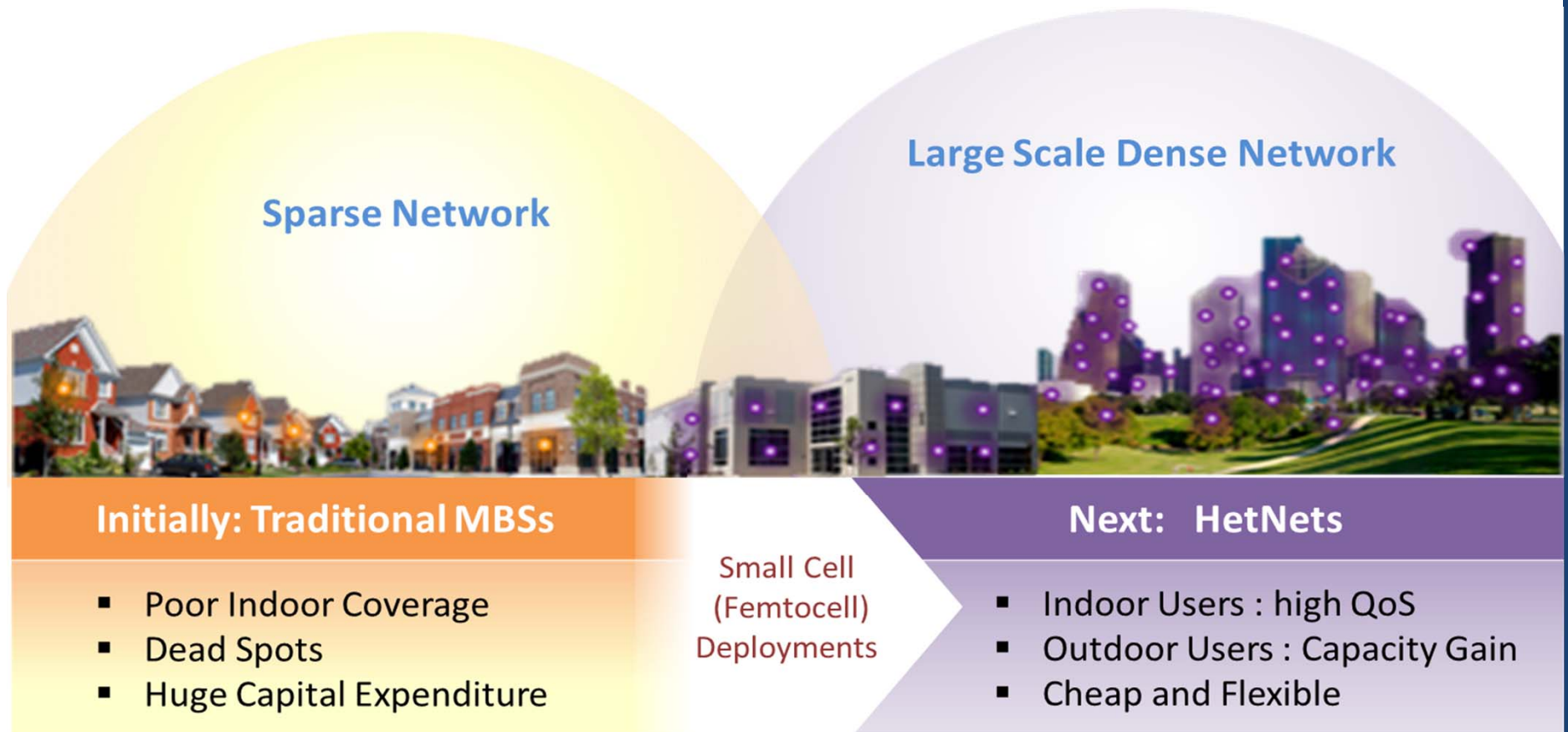
**Capacity** = **Bandwidth (Hz)** x **Spectral Efficiency (bps/Hz)** x **# Links**

**1000** = **10** x **5** x **20**



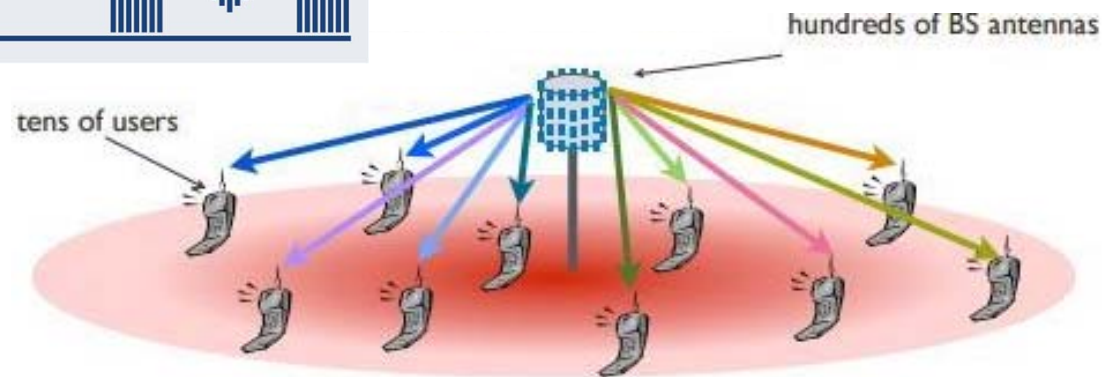
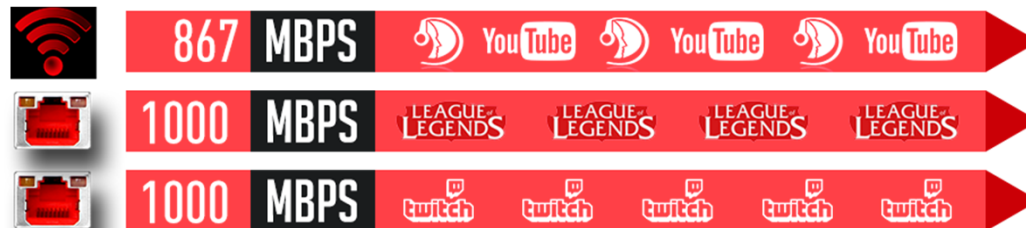
# Background and Motivation

## ❖ Ultra dense networks



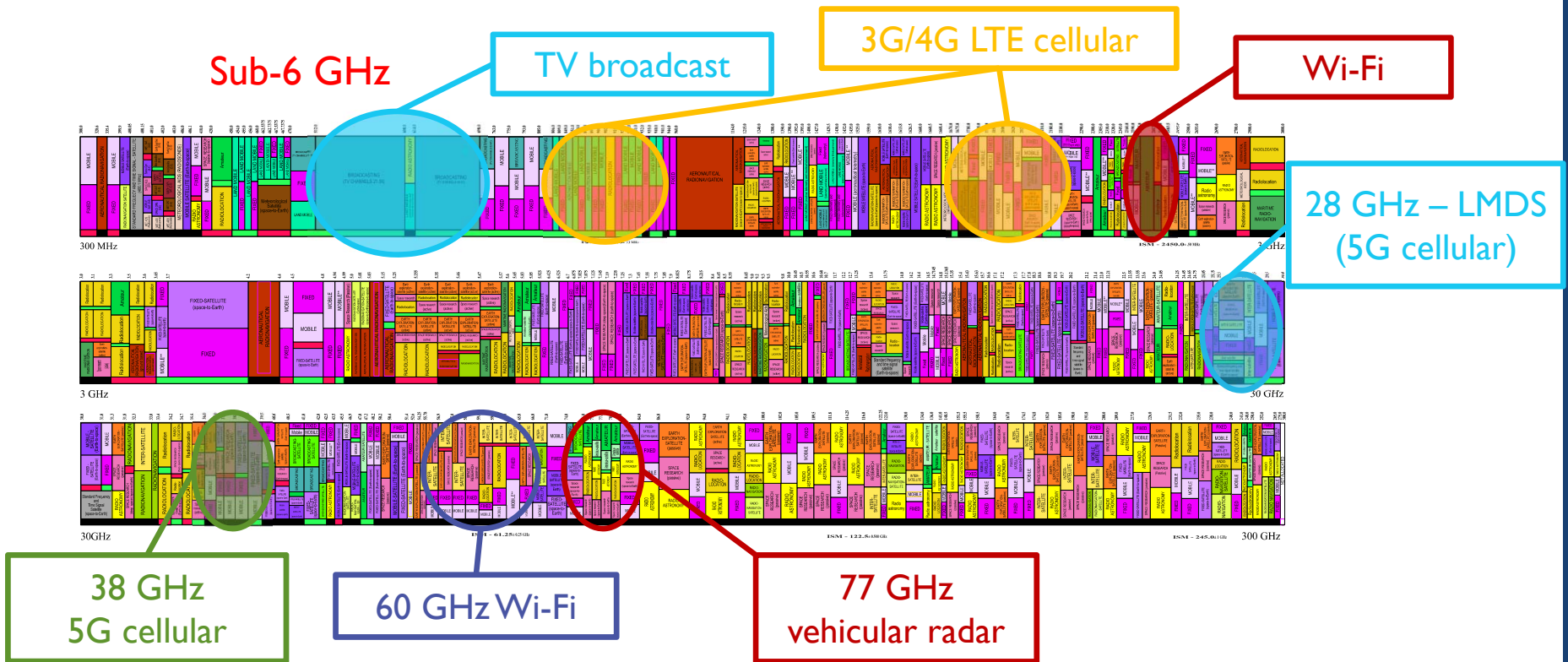
# Background and Motivation

## ❖ Higher spectral efficiency



# Background and Motivation

## ❖ Spectrum crunch: A fundamental bottleneck

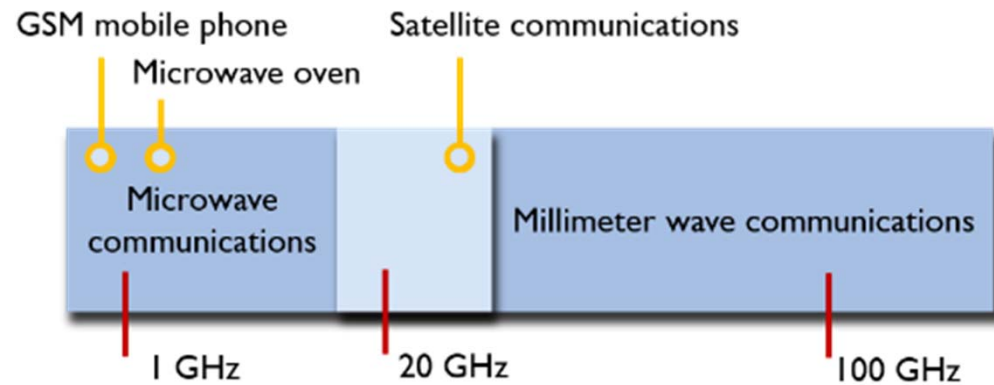


[U.S. Frequency Allocation Chart as of October 2011]



# Background and Motivation

## ❖ New Spectrum: Beyond sub-6 GHz



**5G = Millimeter wave**

At least to someone



# Background and Motivation

## ❖ Latest activities at mm-wave bands



Standardization  
(IEEE 802.11 ad)



Hardware products



Channel models



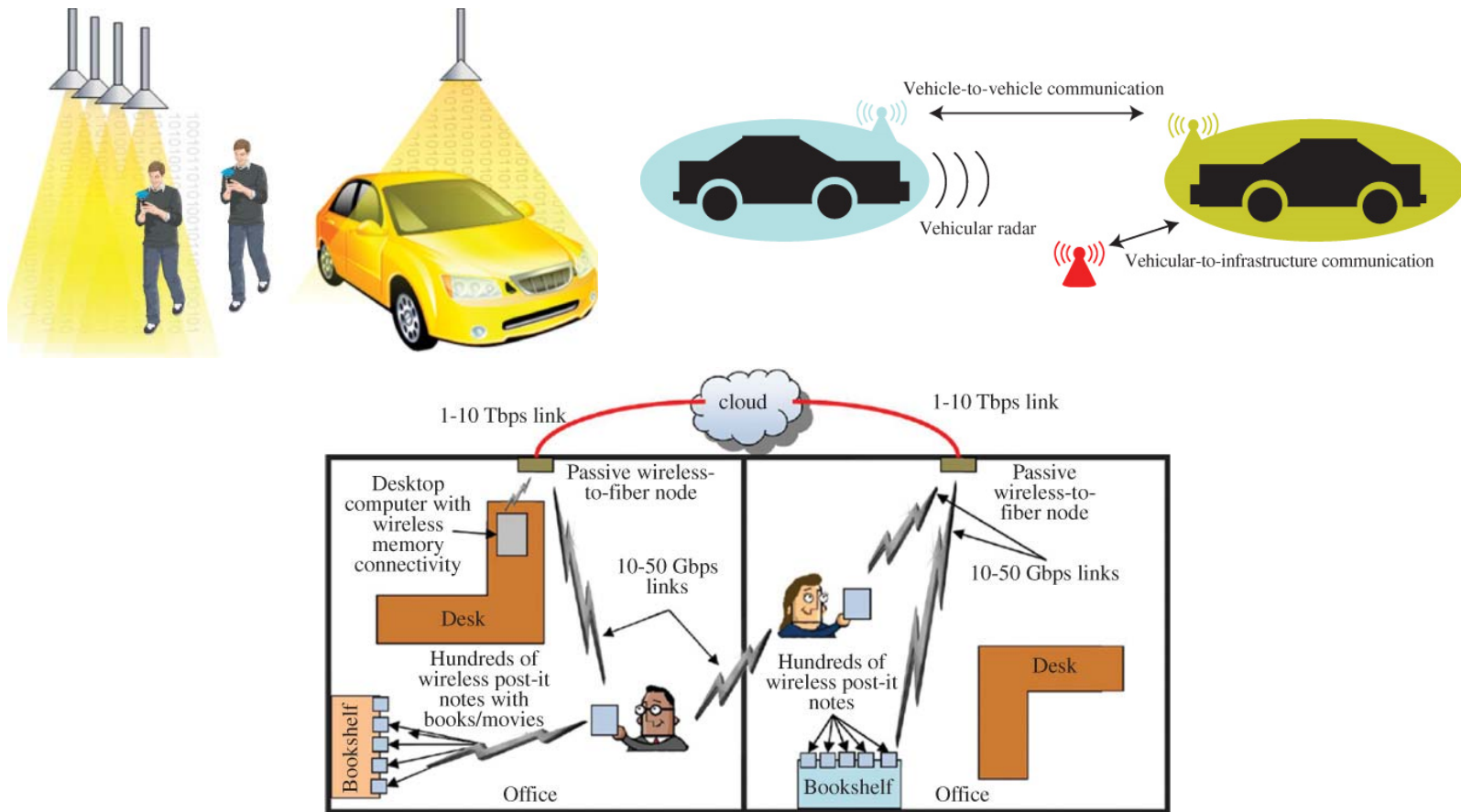
Small cell networks



mm-Wave trial

# Background and Motivation

## ❖ Emerging mm-wave applications [T. S. Rappaport *et al.*, 2014]



# Background and Motivation



Sub-6 GHz signals

$$\text{Receive power: } P_r = \frac{P_t}{4\pi d^2} \frac{\lambda^2}{4\pi}$$



$$\text{Noise power: } N_0 = kT_e B$$

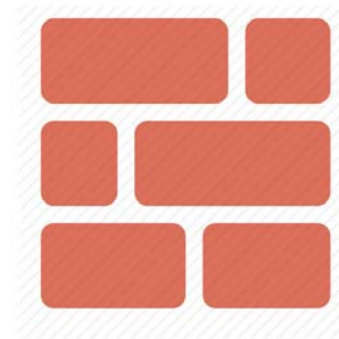


SNR 



Huge path loss

Mm-wave signals

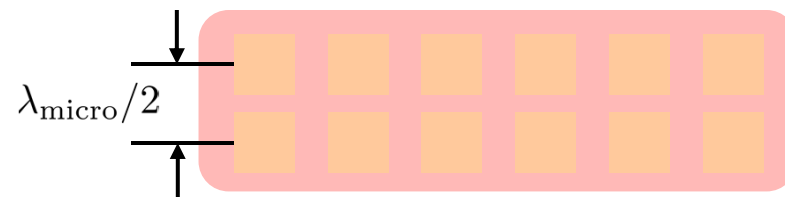
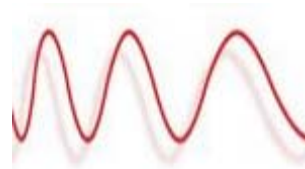


Sensitivity to blockages

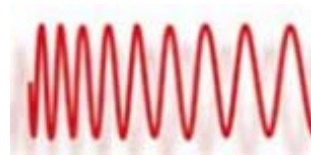
# Background and Motivation



microwave



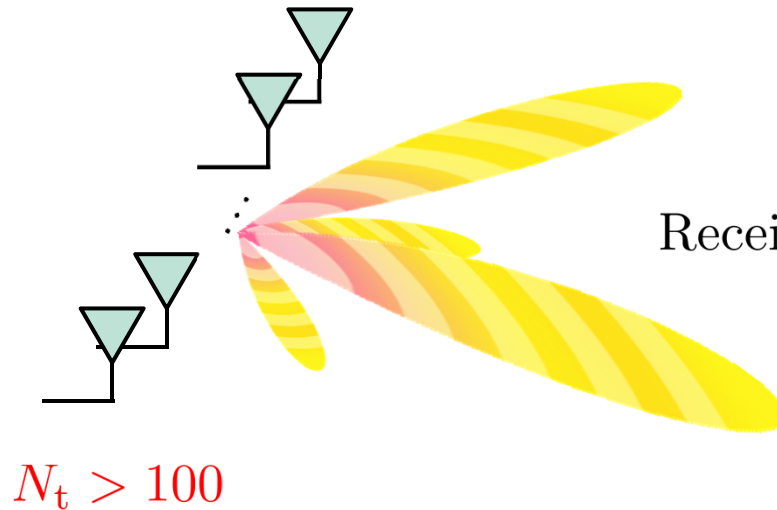
mm-wave



**Small wavelength**  **Large-scale antenna arrays**

More antennas can be patched in a small area

# Background and Motivation



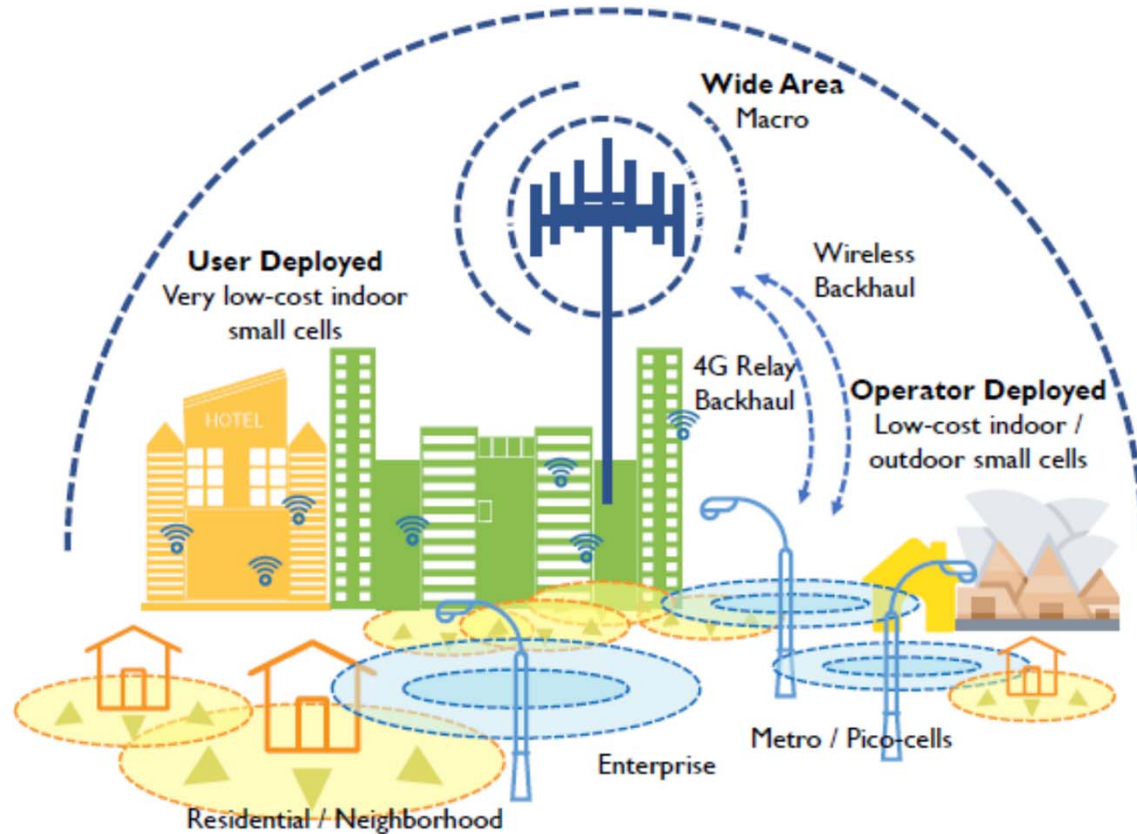
Receive power:  $P_r = \frac{P_t}{4\pi d^2} \frac{\lambda^2}{4\pi} G_t G_r$



## Beamforming!

Higher antenna gains and narrower beams

# Background and Motivation

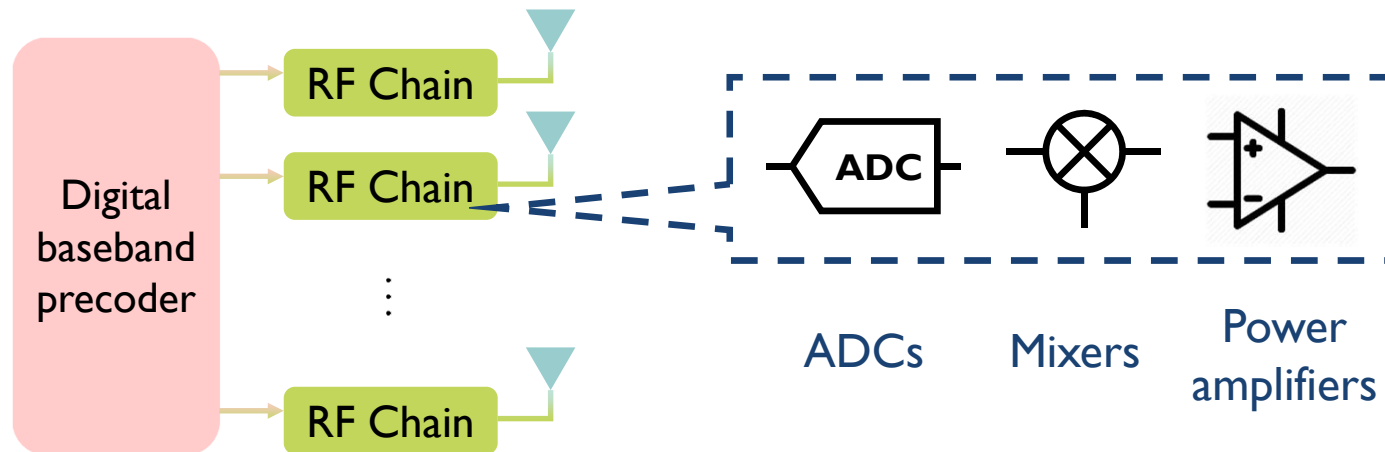


Network densification reduces propagation distance

# Background and Motivation

## ❖ Conventional beamforming

- Performed **digitally** at the **baseband**
- Require **an RF chain per antenna element**



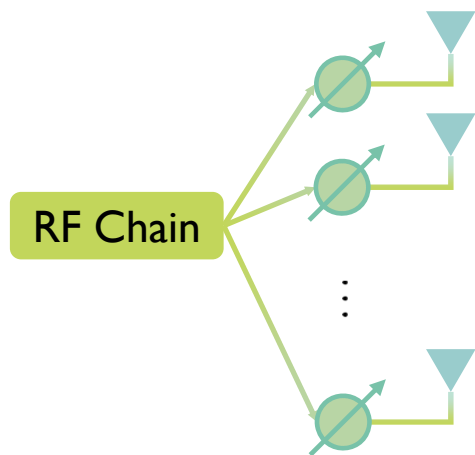
**Costly and power hungry** for large-scale antenna arrays, especially at **mm-wave** bands!



# Background and Motivation

## ❖ Existing solution: **Analog** beamforming

### ➤ **One** RF chain only



$$\mathbf{f}(\varphi) = \frac{1}{\sqrt{N_t}} \left[ 1, \dots, e^{j2\pi k\varphi}, \dots, e^{j2\pi(N_t-1)\varphi} \right]^T$$

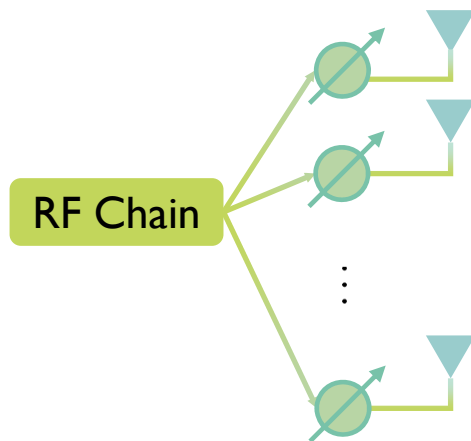
↑  
the decisive variable

- Beams direction readily controlled by a series of **phase shifters** in the **RF domain**
- Low cost and hardware complexity

# Background and Motivation

## ❖ Existing solution: **Analog** beamforming

### ➤ Limitations



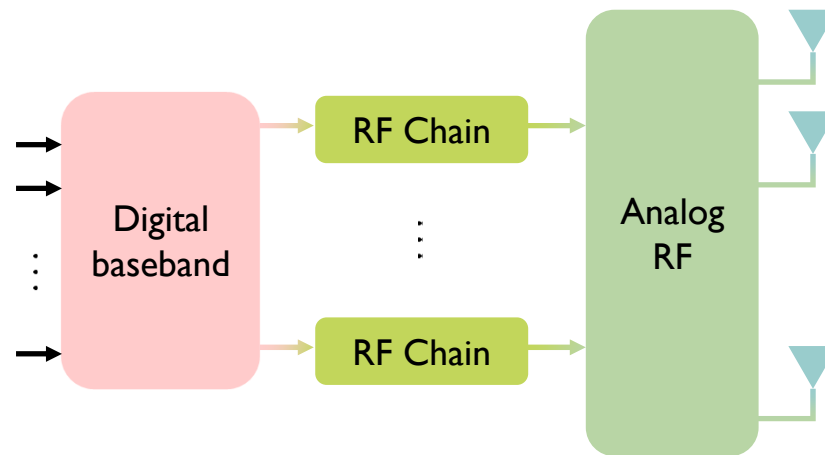
### Benefits of MIMO

- Spatial multiplexing
- Support space-division multiple access (SDMA)

**Analog beamforming can only support single-stream transmissions**

# Background and Motivation

## ❖ Hybrid Beamforming



- Multi-stream transmission, ability to support SDMA
- Multiple RF chains, the **number should be very small**
- Combine the benefits of **digital and analog** beamforming

## ❖ General references on mm-wave

- T. S. Rappaport *et al.*, “Millimeter wave mobile communications for 5G Cellular: It Will Work!,” *IEEE Access*, vol. 1, pp. 335-349, 2013.
- Z. Pi and F. Khan, “An introduction to millimeter-wave mobile broadband systems,” *IEEE Commun. Mag.*, vol. 49, no. 6, pp. 101-107, June 2011.
- E. Torkildson, U. Madhow, and M. Rodwell, “Indoor millimeter wave MIMO: Feasibility and performance,” *IEEE Trans. Wireless Commun.*, vol. 10, no. 12, pp. 4150–4160, Dec. 2011.
- M. R. Akdeniz *et al.*, “Millimeter wave channel modeling and cellular capacity evaluation,” *IEEE J. Sel. Areas Commun.*, vol. 32, no. 6, pp. 1164–1179, Jun. 2014.
- T. S. Rappaport, R. W. Heath, R. C. Daniels, and J. N. Murdock, *Millimeter Wave Wireless Communications*. New York, NY, USA: Pearson Education, 2014.
- P. Wang, Y. Li, L. Song, and B. Vucetic, “Multi-gigabit millimeter wave wireless communications for 5G: From fixed access to cellular networks,” *IEEE Commun. Mag.*, vol. 53, no. 1, pp. 168–178, Jan. 2015.
- S. Rangan, T. S. Rappaport, and E. Erkip, “Millimeter-wave cellular wireless networks: Potentials and challenges,” *Proc. IEEE*, vol. 102, no. 3, pp. 366–385, Feb. 2014.

## ❖ Recognitions on hybrid beamforming

- O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, Jr., “Spatially sparse precoding in millimeter wave MIMO systems,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499-1513, Mar. 2014.
  - **The 2017 Marconi Prize Paper Award in Wireless Communications**
- F. Sofrabi and W. Yu, “Hybrid digital and analog beamforming design for large-scale antenna arrays,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 501-513, Apr. 2016.
  - **The 2017 IEEE Signal Processing Society Best Paper Award**
- A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., “Channel estimation and hybrid precoding for millimeter wave cellular systems,” *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831-846, Oct. 2014.
  - **The 2016 Signal Processing Society Young Author Best Paper Award**
- X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 485-500, Apr. 2016.
  - **The 2018 Signal Processing Society Young Author Best Paper Award**

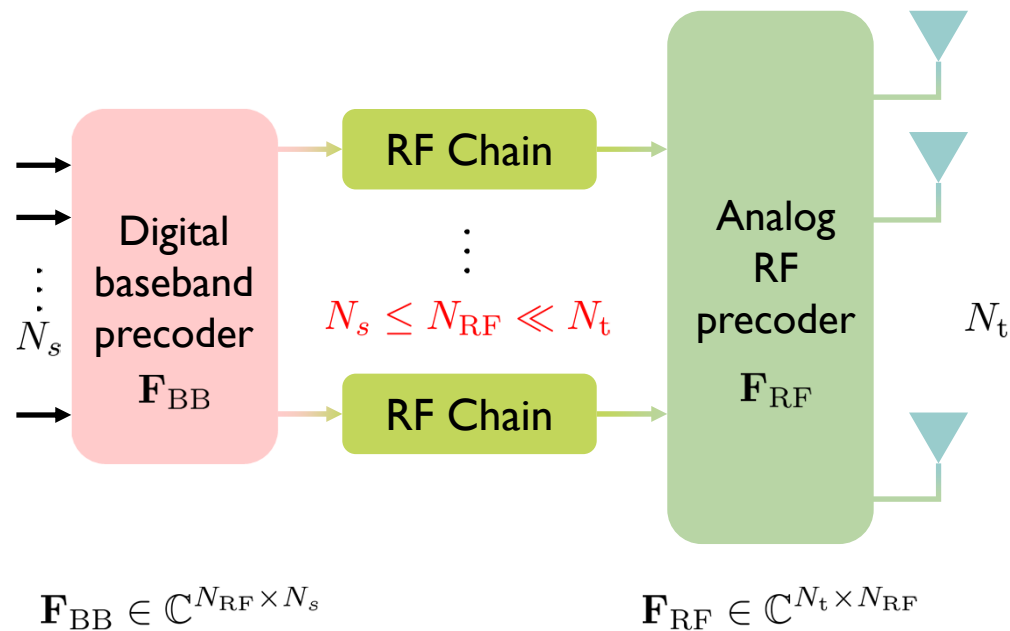
# Preliminaries of Hybrid Beamforming

# Preliminaries of Hybrid Beamforming

## ❖ Hybrid beamforming

➤ Also called *Hybrid precoding*; *Analog/digital precoding*

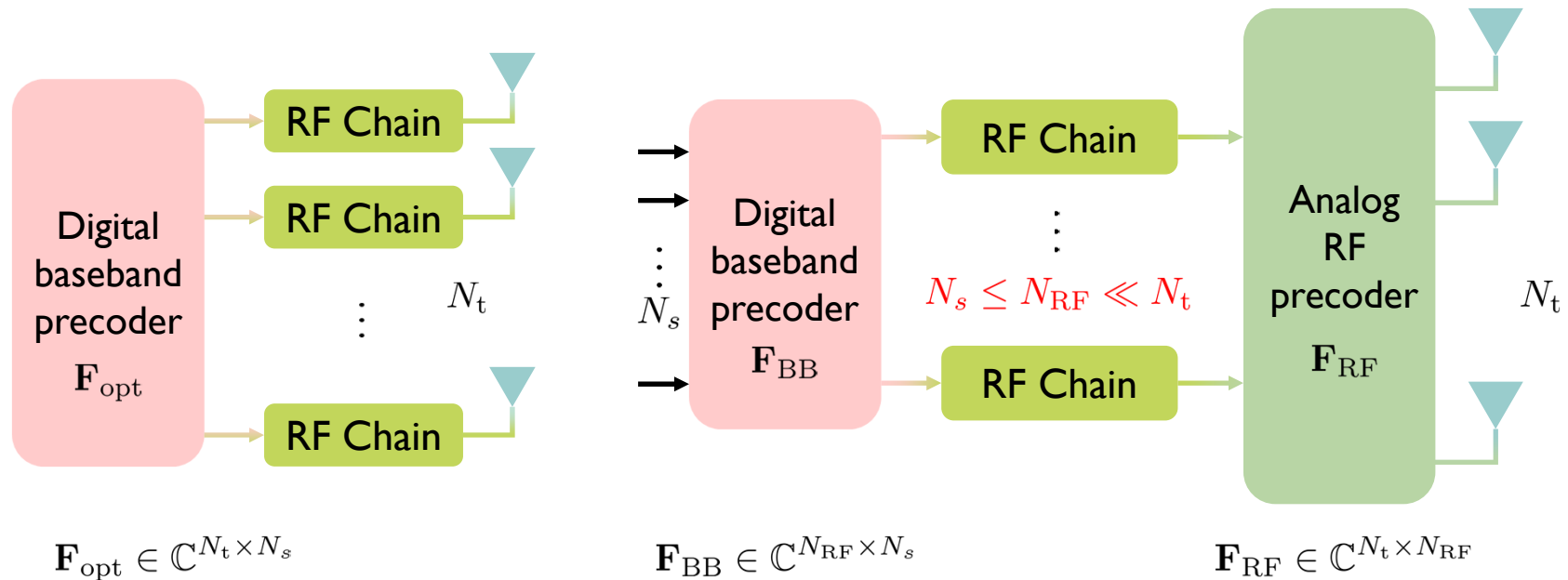
➤ **Notations** in hybrid beamforming





# Preliminaries of Hybrid Beamforming

## ❖ Fully digital precoding vs. Hybrid precoding



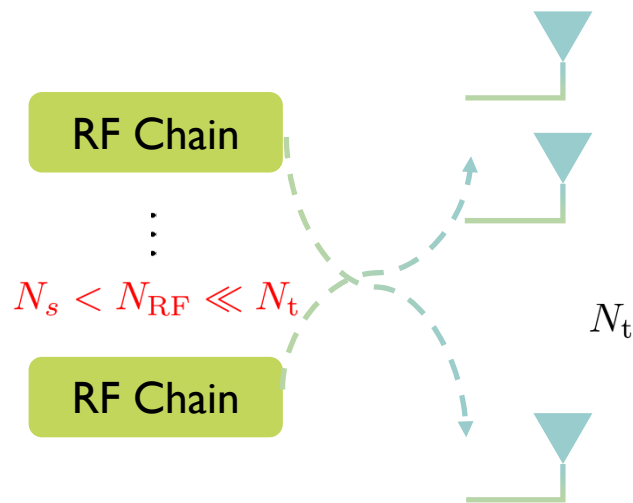
- Main differentiating part: **Analog RF precoder**
- Mapping from low-dimensional RF chains to high-dimensional antennas, typically implemented by **phase shifters**

# Preliminaries of Hybrid Beamforming

## ❖ Hybrid precoder structure

### (I) Mapping strategy:

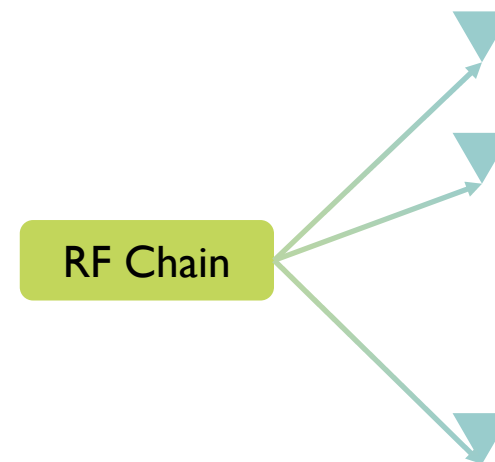
Which antennas should be connected to each RF chain?



**Signal flow**

### (II) Hardware implementation:

What kind of hardware should be used to realize each connection?

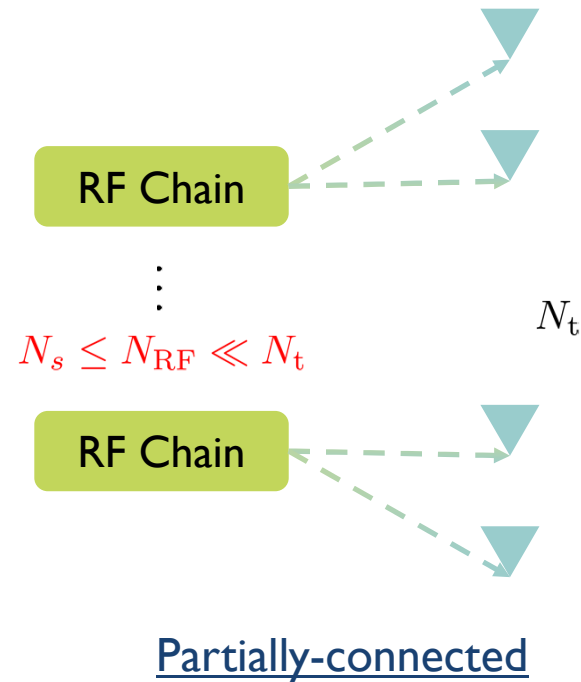
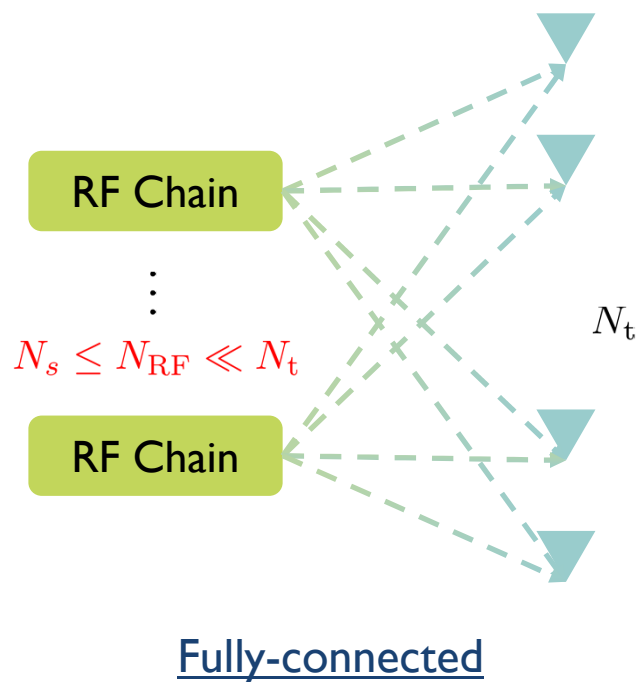


**Adopted hardware**

# Preliminaries of Hybrid Beamforming

## ❖ The state-of-the-art hybrid precoder structure

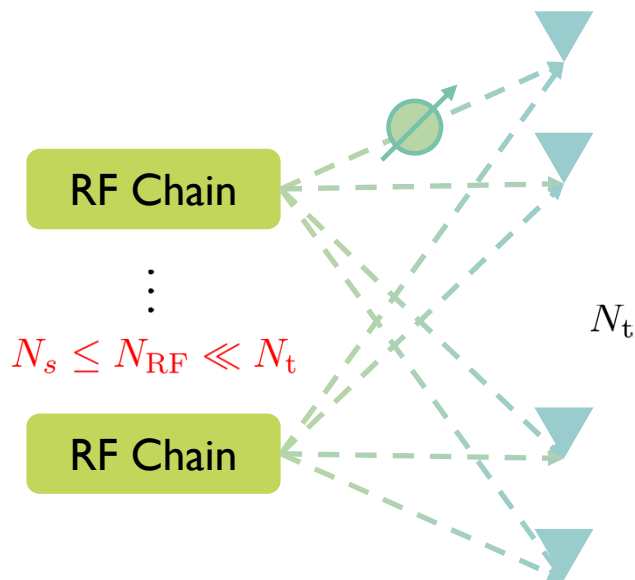
➤ Mainly focus on different mapping strategies



# Preliminaries of Hybrid Beamforming

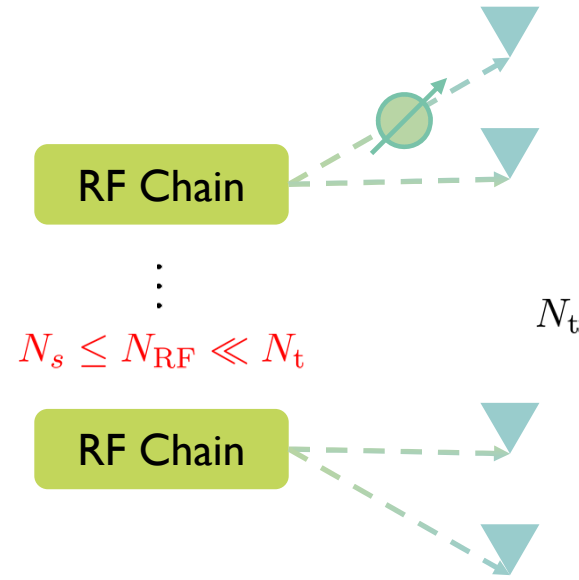
## ❖ The state-of-the-art hybrid precoder structure

- One prevalent hardware implementation: **Single phase shifter (SPS)**



SPS Fully-connected

$$N_{PS} = N_t N_{RF}$$

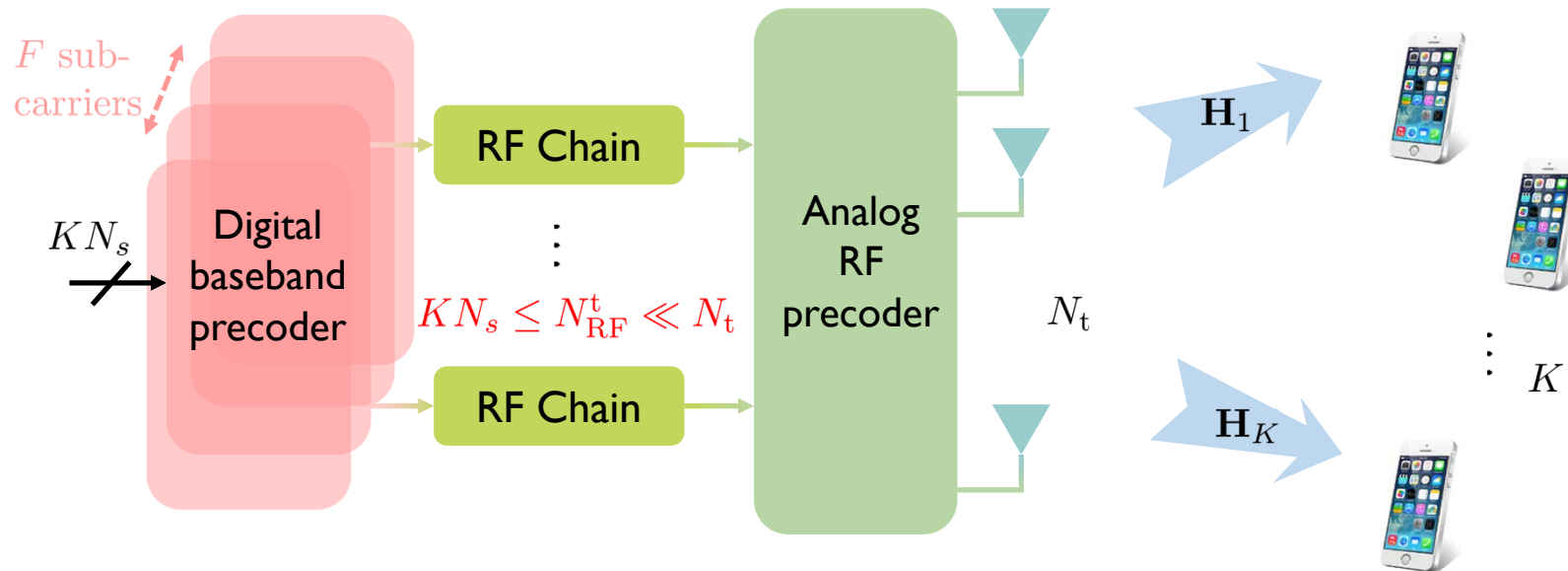


SPS Partially-connected

$$N_{PS} = N_t$$

# Preliminaries of Hybrid Beamforming

## ❖ General multiuser multicarrier (MU-MC) systems

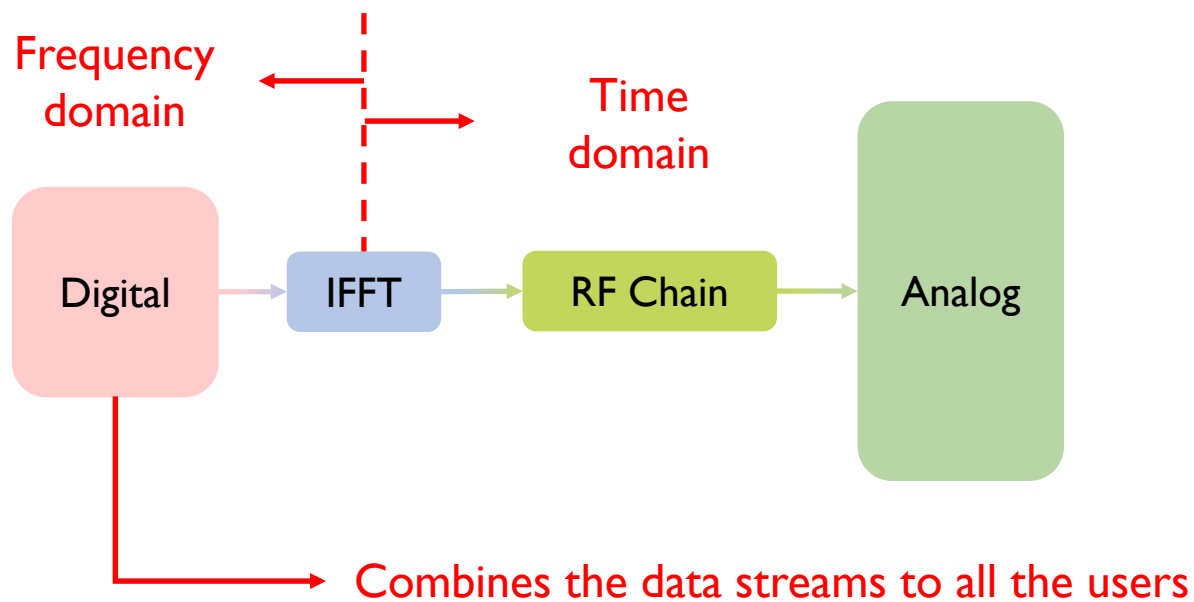


- One single digital precoder for each user on each subcarrier

$$\mathbf{F}_{BBk,f}$$

# Preliminaries of Hybrid Beamforming

## ❖ General multiuser multicarrier (MU-MC) systems



➤ Analog precoder  $\mathbf{F}_{\text{RF}}$  is shared by all the users and subcarriers

## ❖ Generic hybrid precoder design problem

- Minimize the Euclidean distance between the hybrid precoders and the fully digital precoder [O. El Ayach *et al.*, 2014]

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && \|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \leq P_{\text{max}} \end{aligned}$$

$$\mathbf{F}_{\text{RF}} \in \mathcal{A}_x \quad \text{Main difficulty}$$

$$\mathbf{F}_{\text{opt}} = \left[ \mathbf{F}_{\text{opt}_{1,1}}, \dots, \mathbf{F}_{\text{opt}_{k,f}}, \dots, \mathbf{F}_{\text{opt}_{K,F}} \right] \in N_t \times KN_s F$$

$$\mathbf{F}_{\text{BB}} = \left[ \mathbf{F}_{\text{BB}_{1,1}}, \dots, \mathbf{F}_{\text{BB}_{k,f}}, \dots, \mathbf{F}_{\text{BB}_{K,F}} \right] \in N_{\text{RF}}^t \times KN_s F$$

- $\mathcal{A}_x$  varies according to different hybrid precoder structures, e.g.,  $|(\mathbf{F}_{\text{RF}})_{i,j}| = 1$  for the SPS fully-connected structure



# Preliminaries of Hybrid Beamforming

## ❖ Generic hybrid precoder design problem

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && \|\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \leq P_{\text{max}} \\ & && \mathbf{F}_{\text{RF}} \in \mathcal{A}_x \end{aligned}$$

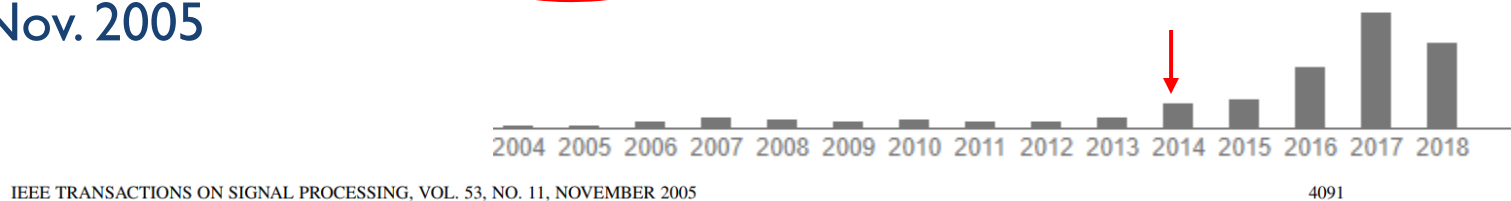
- This formulation applies for an arbitrary digital precoder
- It is applicable for different hybrid beamformer structures
- It facilitates beamforming algorithm design

# Preliminaries of Hybrid Beamforming

## ❖ An early work on hybrid beamforming

Cited by 326

➤ Nov. 2005



## Variable-Phase-Shift-Based RF-Baseband Codesign for MIMO Antenna Selection

Xinying Zhang, Andreas F. Molisch, *Fellow, IEEE*, and Sun-Yuan Kung, *Fellow, IEEE*

- Phase shifter based RF beamforming
- $N_{RF}=2$  is enough for  $N_s=1$  to achieve the performance of the fully digital precoder
- Have not got too much attention before hybrid beamforming was proposed (cited 75 times before 2014 while 268 times after 2014)

## ❖ An extension

➤ Sep. 2014

## On Achieving Optimal Rate of Digital Precoder by RF-Baseband Codesign for MIMO Systems

Edin Zhang and Chiachi Huang  
Department of Communications Engineering  
Yuan Ze University  
Taoyuan, Taiwan

- **Generalization:**  $N_{RF}=2N_s$  to achieve the performance of the fully digital precoder
- The number of RF chains to achieve fully digital will be very large for MU-MC systems

# Preliminaries of Hybrid Beamforming

## ❖ Questions to be answered in this tutorial

➤ **Q1:** Can hybrid precoder provide performance close to the fully digital one with  $N_{\text{RF}} < 2N_s$ ?

Spectral efficiency

➤ **Q2:** How many RF chains are needed?

➤ **Q3:** How many phase shifters are needed?

Hardware efficiency

➤ **Q4:** How to connect RF chains with antennas?

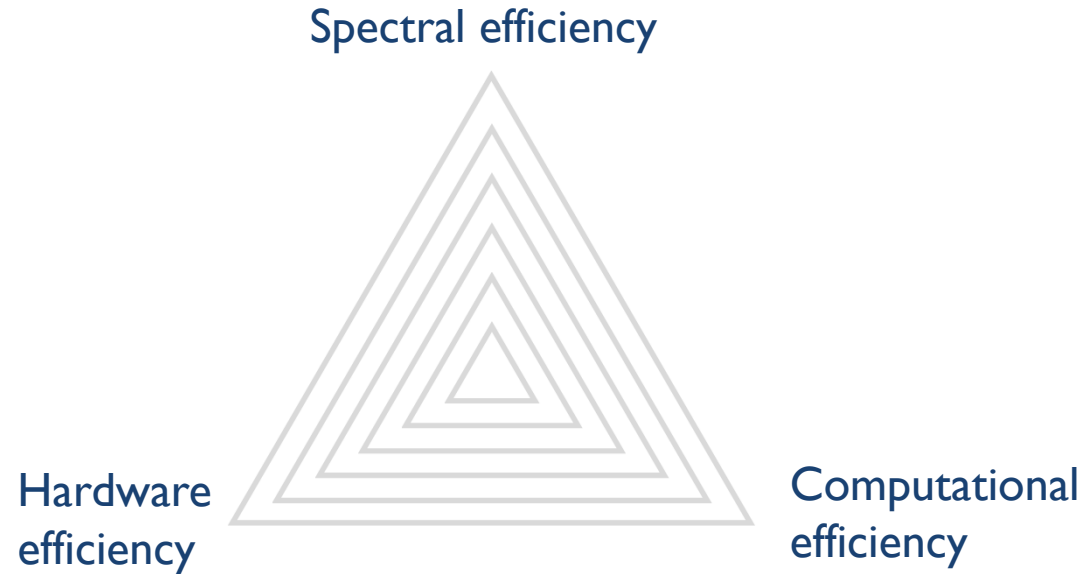
➤ **Q5:** How to efficiently design hybrid precoding algorithms?

Computational efficiency

# Preliminaries of Hybrid Beamforming

## ❖ Performance metrics

### ➤ “Scoring triangle”

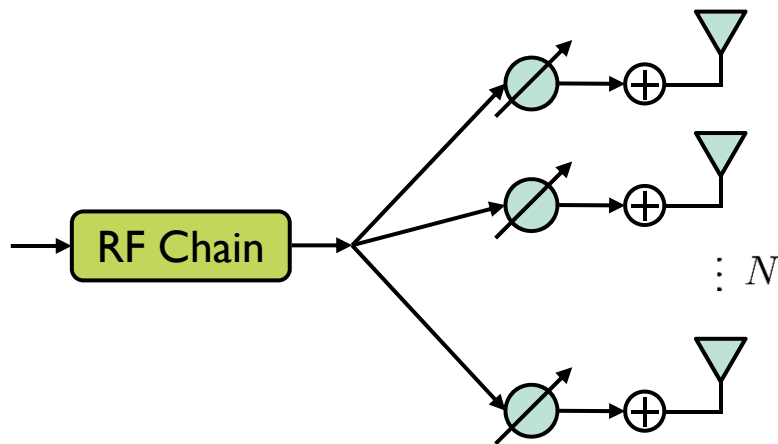


# Improve Spectral Efficiency: Approaching the Fully Digital Beamforming

[Ref] X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems,” *IEEE J. Sel. Topics Signal Process., Special Issue on Signal Process. for Millimeter Wave Wireless Commun.*, vol. 10, no. 3, pp. 485-500, Apr. 2016. (**The 2018 IEEE Signal Processing Society Young Author Best Paper Award**)

# Improve Spectral Efficiency

## ❖ Single phase shifter (SPS) implementation

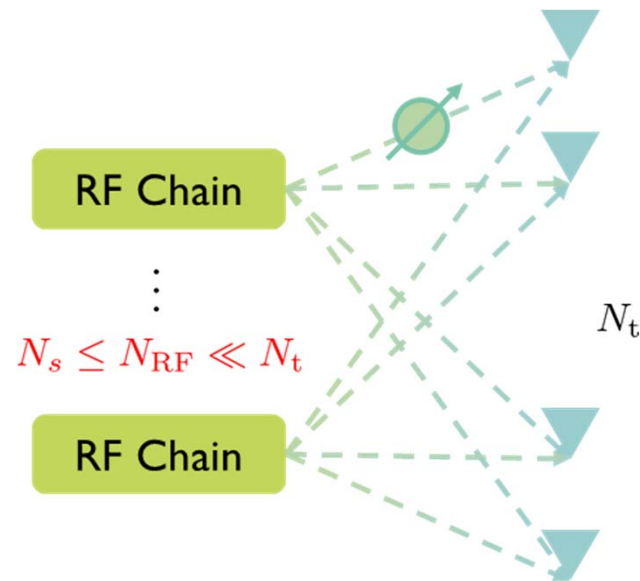


$$N = \begin{cases} N_t & \text{fully-connected} \\ N_t/N_{\text{RF}}^t & \text{partially-connected} \end{cases}$$

➤ Fully digital achieving condition:  $N_{\text{RF}}^t = 2KN_s$ ,  $N_{\text{RF}}^r = 2N_s$

**Q: Can we further reduce the number of RF chains?**

## (I) Fully-Connected Mapping





## ❖ Existing work

➤ Mar. 2014

Citation >1016

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 13, NO. 3, MARCH 2014

1499

## Spatially Sparse Precoding in Millimeter Wave MIMO Systems

Omar El Ayach, *Member, IEEE*, Sridhar Rajagopal, *Senior Member, IEEE*, Shadi Abu-Surra, *Member, IEEE*,  
Zhouyue Pi, *Senior Member, IEEE*, and Robert W. Heath, Jr., *Fellow, IEEE*

- Orthogonal matching pursuit (OMP) algorithm
- The columns of the analog precoding matrix  $\mathbf{F}_{\text{RF}}$  is selected from a candidate set  $\mathcal{C}$  (array response vectors)

$$\mathcal{C} = \{\mathbf{f}(\varphi_i)\}_{i=1}^{|\mathcal{C}|} \quad \mathbf{f}(\varphi_i) = \frac{1}{\sqrt{N_t}} [1, \dots, e^{j2\pi k\varphi_i}, \dots, e^{j2\pi(N_t-1)\varphi_i}]^T$$

# Improve Spectral Efficiency

## ❖ Existing work

### ➤ OMP Algorithm

---

**Algorithm 1** Spatially Sparse Precoding via Orthogonal Matching Pursuit

---

**Require:**  $\mathbf{F}_{\text{opt}}$

1:  $\mathbf{F}_{\text{RF}} = \text{Empty Matrix}$

2:  $\mathbf{F}_{\text{res}} = \mathbf{F}_{\text{opt}}$

3: **for**  $i \leq N_t^{\text{RF}}$  **do**

4:  $\mathbf{\Psi} = \mathbf{A}_t^* \mathbf{F}_{\text{res}}$

5:  $k = \arg \max_{\ell=1, \dots, N_{\text{cl}} N_{\text{ray}}} (\mathbf{\Psi} \mathbf{\Psi}^*)_{\ell, \ell}$

6:  $\mathbf{F}_{\text{RF}} = \left[ \mathbf{F}_{\text{RF}} \mid \mathbf{A}_t^{(k)} \right]$

7:  $\mathbf{F}_{\text{BB}} = (\mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{RF}})^{-1} \mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{opt}}$

8:  $\mathbf{F}_{\text{res}} = \frac{\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}}{\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F}$

9: **end for**

10:  $\mathbf{F}_{\text{BB}} = \sqrt{N_s} \frac{\mathbf{F}_{\text{BB}}}{\|\mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F}$

11: **return**  $\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}$

---

Find the array response vector along which the optimal precoder has the maximum projection

Appends the selected array response vector to the  $\mathbf{F}_{\text{RF}}$

Least squares solution to  $\mathbf{F}_{\text{BB}}$

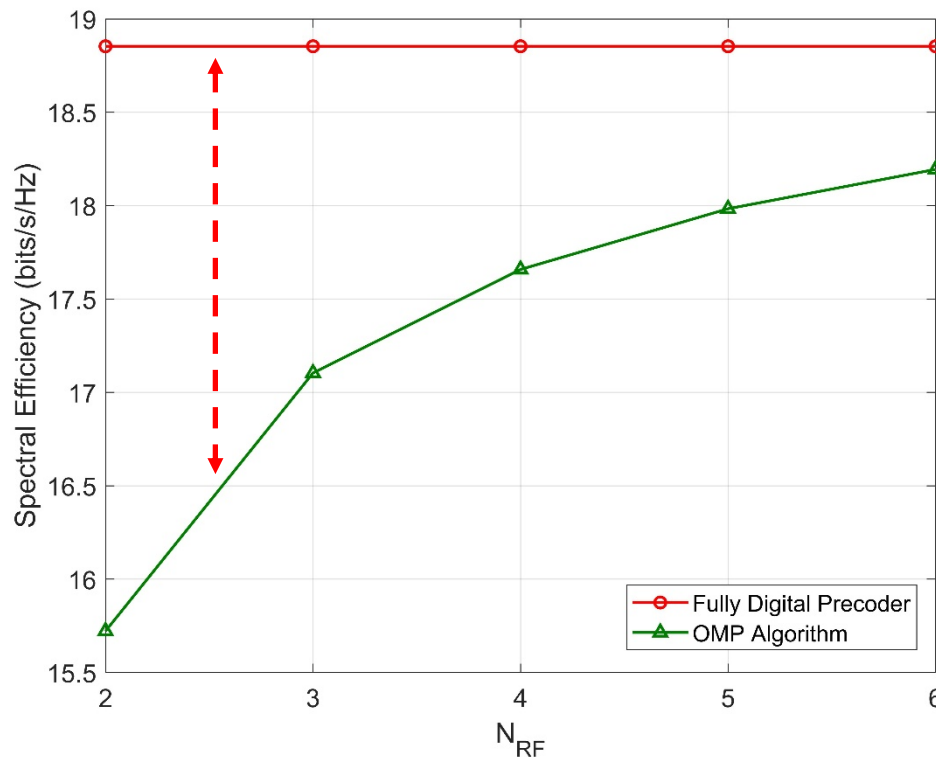
Calculate “residual precoding matrix”

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Simulation result

$$N_t = 144, N_r = 36, N_{\text{RF}}^t = N_{\text{RF}}^r = N_{\text{RF}}, N_s = 2, \text{SNR} = 0 \text{ dB}$$



➤ Prominent performance loss especially with a small number of RF chains

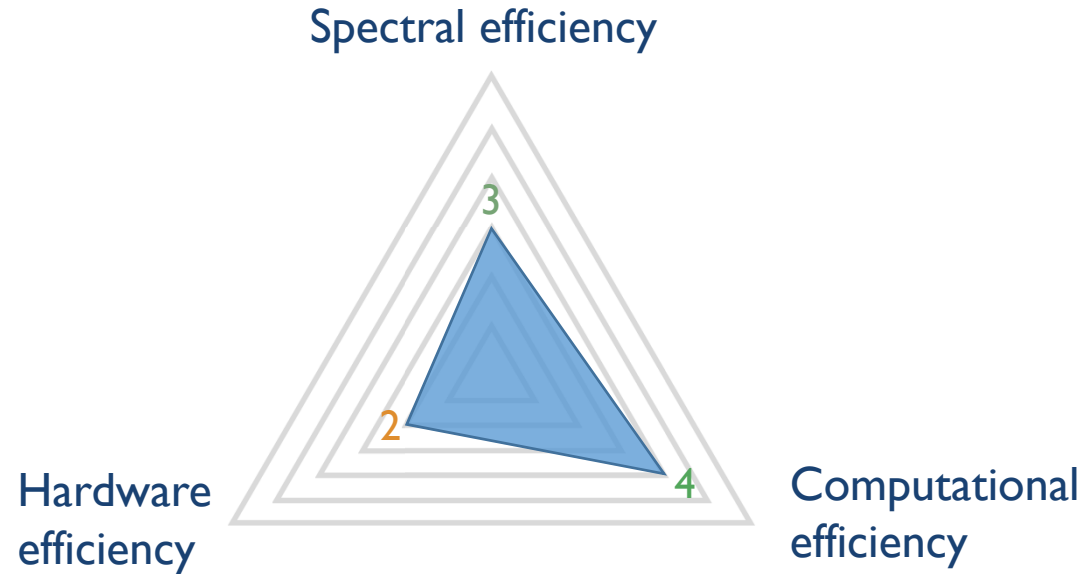
Q: How to improve spectral efficiency with a few RF chains?

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Performance metrics

#### ➤ “Scoring triangle”



**Baseline: SPS fully-connected with OMP**

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Start from single-user systems

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

### ➤ Alternating minimization

$$\underset{\mathbf{F}_{\text{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F^2$$

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

### ➤ Digital precoder: $\mathbf{F}_{\text{BB}} = \mathbf{F}_{\text{RF}}^\dagger \mathbf{F}_{\text{opt}}$

### ➤ Difficulty: Analog precoder design with the **unit modulus constraints**

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

### ➤ The vector $\mathbf{x} = \text{vec}(\mathbf{F}_{\text{RF}})$ forms a complex circle manifold

$$\mathcal{M}_{cc}^m = \{\mathbf{x} \in \mathbb{C}^m : |\mathbf{x}_1| = |\mathbf{x}_2| = \cdots = |\mathbf{x}_m| = 1\}, \quad m = N_t N_{\text{RF}}^t.$$

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization

#### ➤ What is a manifold?



- In mathematics, a **manifold** is a topological space that **locally resembles Euclidean space near each point**. More precisely, each point of an  $n$ -dimensional manifold has a neighborhood that is homeomorphic to the Euclidean space of dimension  $n$ .

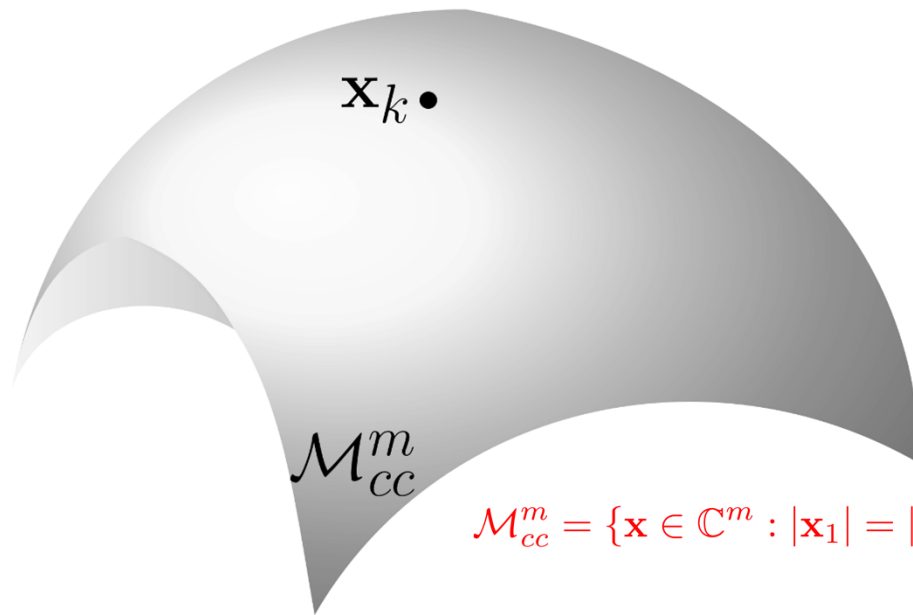
#### ➤ How to optimize on manifolds?

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization (cont.)

- Euclidean space: **gradient descent**
- Similar approaches on manifolds?



**Q:** For any given point  $\mathbf{x}_k$  on the manifold, where to go to further decrease the objective?

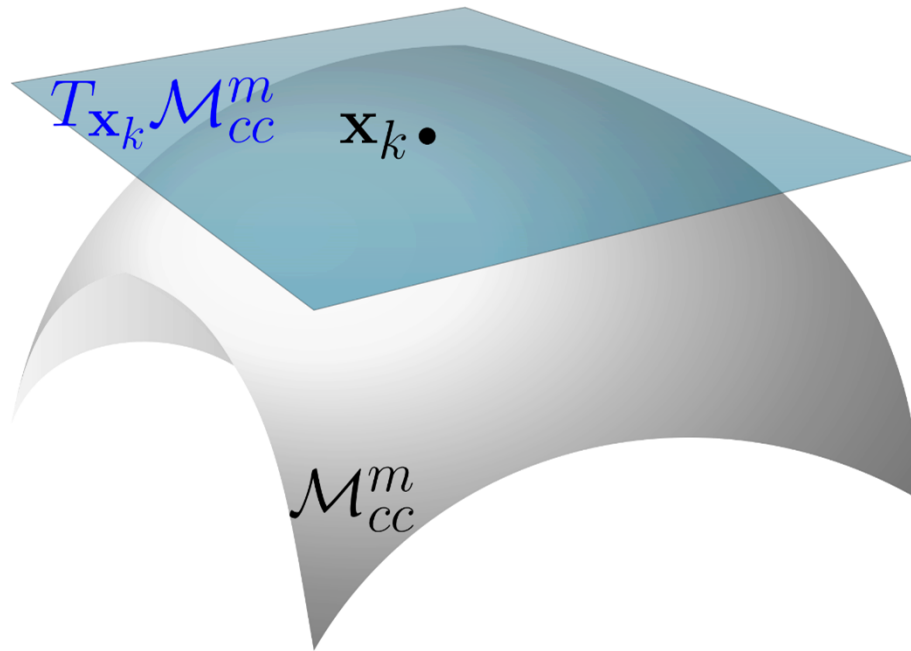
$$\mathcal{M}_{cc}^m = \{\mathbf{x} \in \mathbb{C}^m : |\mathbf{x}_1| = |\mathbf{x}_2| = \dots = |\mathbf{x}_m| = 1\}, \quad m = N_t N_{RF}^t.$$

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization (cont.)

**Tangent space:** Contains all possible directions that tangentially pass through  $\mathbf{x}_k$



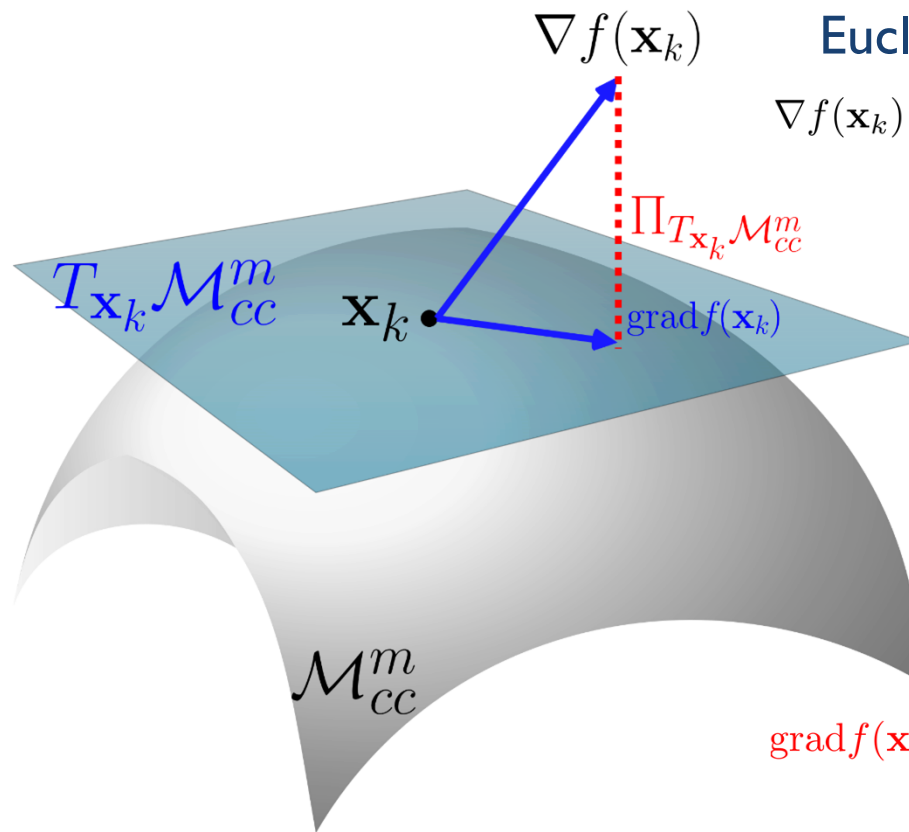
**Q:** How to find the direction with the steepest descend?



# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization (cont.)



Euclidean gradient is easy to calculate

$$\nabla f(\mathbf{x}_k) = -2(\mathbf{F}_{\text{BB}}^* \otimes \mathbf{I}_{N_t}) [\text{vec}(\mathbf{F}_{\text{opt}}) - (\mathbf{F}_{\text{BB}}^T \otimes \mathbf{I}_{N_t}) \mathbf{x}_k]$$

The **Riemannian gradient** at  $\mathbf{x}_k$  is a tangent vector  $\text{grad}f(\mathbf{x}_k)$  given by the orthogonal projection of the Euclidean gradient  $\nabla f(\mathbf{x}_k)$  onto the tangent space

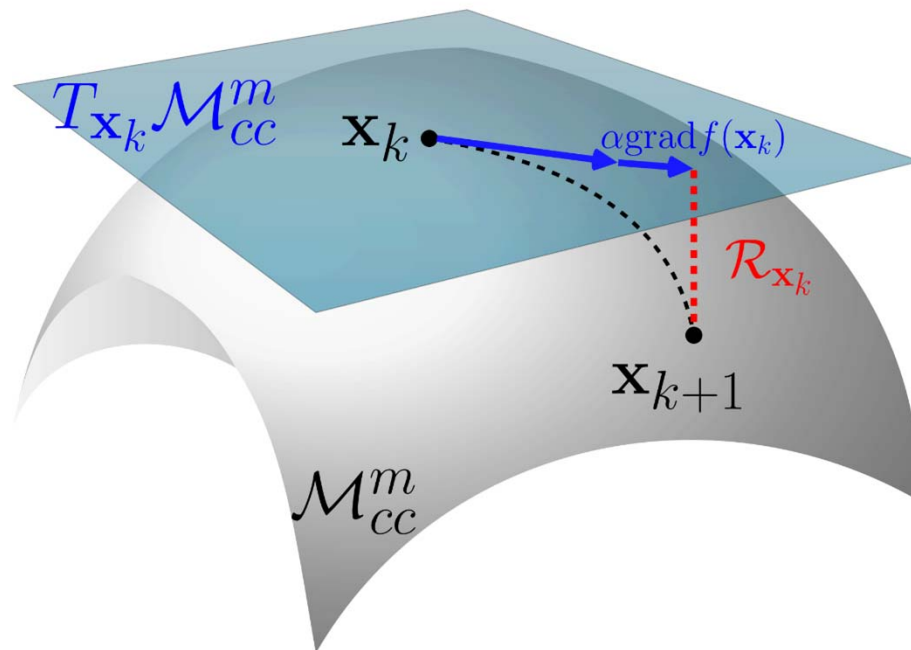
$$\begin{aligned} \text{grad}f(\mathbf{x}_k) &= \text{Proj}_{\mathbf{x}_k} \nabla f(\mathbf{x}_k) \\ &= \nabla f(\mathbf{x}_k) - \Re\{\nabla f(\mathbf{x}_k) \circ \mathbf{x}_k^*\} \circ \mathbf{x}_k \end{aligned}$$

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization (cont.)

After determining the step size, the destination is not on the manifold



**Retraction:** mapping from tangent vectors back to the manifold itself

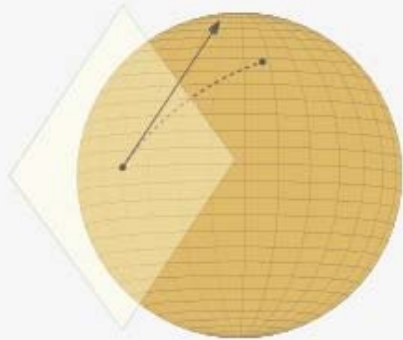
$$\text{Retr}_{\mathbf{x}_k} : T_{\mathbf{x}_k} \mathcal{M}_{CC}^m \rightarrow \mathcal{M}_{CC}^m :$$

$$\alpha \mathbf{d} \mapsto \text{Retr}_{\mathbf{x}_k}(\alpha \mathbf{d}) = \text{vec} \begin{bmatrix} (\mathbf{x} + \alpha \mathbf{d})_i \\ |(\mathbf{x} + \alpha \mathbf{d})_i| \end{bmatrix}$$

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ Manifold optimization (cont.)



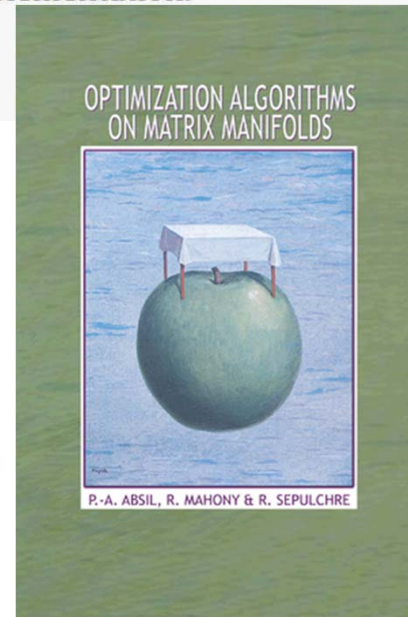
#### **Manopt: a Matlab toolbox for optimization on Manifolds**

Manopt, available at [manopt.org](http://manopt.org), is a user-friendly, open source and **documented** Matlab toolbox which can be used to leverage the power of modern Riemannian optimization algorithms with ease. Manopt won the **ORBEL Wolsey Award 2014** for best open source operational research implementation.

[Tell me more/less](#)

<https://www.manopt.org/>

**ORBEL Wolsey Award 2014**



# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ MO-AltMin Algorithm

---

#### MO-AltMin Algorithm: Manifold Optimization Based Hybrid Precoding for the Fully-connected Structure

---

**Input:**  $\mathbf{F}_{\text{opt}}$

- 1: Construct  $\mathbf{F}_{\text{RF}}^{(0)}$  with random phases and set  $k = 0$ ;
- 2: **repeat**
- 3: Fix  $\mathbf{F}_{\text{RF}}^{(k)}$ , and  $\mathbf{F}_{\text{BB}}^{(k)} = \mathbf{F}_{\text{RF}}^{(k)\dagger} \mathbf{F}_{\text{opt}}$ ;
- 4: Optimize  $\mathbf{F}_{\text{RF}}^{(k+1)}$  using Algorithm 1 when  $\mathbf{F}_{\text{BB}}^{(k)}$  is fixed;
- 5:  $k \leftarrow k + 1$ ;

Manifold optimization  
for analog precoder



- 6: **until** a stopping criterion triggers;
  - 7: For the digital precoder at the transmit end, normalize  
$$\hat{\mathbf{F}}_{\text{BB}} = \frac{\sqrt{N_s}}{\|\mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F} \mathbf{F}_{\text{BB}}.$$
-

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ SPS fully-connected (cont.)

➤ A low-complexity algorithm

➤ Enforce a semi-orthogonal constraint on  $\mathbf{F}_{\text{BB}}$

$$\mathbf{F}_{\text{BB}}^H \mathbf{F}_{\text{BB}} = \alpha^2 \mathbf{F}_{\text{DD}}^H \mathbf{F}_{\text{DD}} = \alpha^2 \mathbf{I}_{KN_s}$$

$$\|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB}}\|_F^2 \leq \|\mathbf{F}_{\text{opt}}\|_F^2 - 2\alpha \Re \text{Tr}(\mathbf{F}_{\text{DD}} \mathbf{F}_{\text{opt}}^H \mathbf{F}_{\text{RF}}) + \alpha^2 \|\mathbf{F}_{\text{RF}}\|_F^2$$

➤ Digital precoder design

$$\underset{\mathbf{F}_{\text{DD}}}{\text{maximize}} \quad \Re \text{Tr}(\mathbf{F}_{\text{DD}} \mathbf{F}_{\text{opt}}^H \mathbf{F}_{\text{RF}})$$

$$\text{subject to} \quad \mathbf{F}_{\text{DD}}^H \mathbf{F}_{\text{DD}} = \mathbf{I}_{KN_s}$$

➤ Semi-orthogonal Procrustes solution  $\mathbf{F}_{\text{DD}} = \mathbf{V}_1 \mathbf{U}^H$

# Improve Spectral Efficiency

## (I) Fully-Connected Mapping

### ❖ SPS fully-connected (cont.)

#### ➤ Analog precoder design

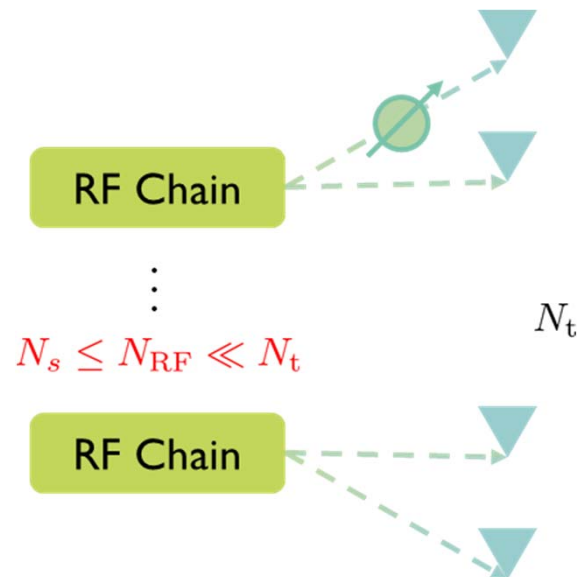
$$\begin{aligned} & \underset{\alpha, \mathbf{F}_{\text{RF}}}{\text{minimize}} && \left\| \Re(\mathbf{F}_{\text{opt}} \mathbf{F}_{\text{DD}}^H) - \alpha \mathbf{F}_{\text{RF}} \right\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

#### ➤ Phase extraction (PE-AltMin)

$$\arg(\mathbf{F}_{\text{RF}}) = \arg(\mathbf{F}_{\text{opt}} \mathbf{F}_{\text{DD}}^H)$$

#### ➤ When $N_{\text{RF}} = N_s$ , the upper bound is tight, the only approximation is the additional semi-orthogonal constraint

## (II) Partially-Connected Mapping



# Improve Spectral Efficiency

## (II) Partially-Connected Mapping

### ❖ Existing work

➤ Apr. 2016

Citation > 227

998

IEEE JOURNAL ON SELECTED AREAS IN COMMUNICATIONS, VOL. 34, NO. 4, APRIL 2016

## Energy-Efficient Hybrid Analog and Digital Precoding for MmWave MIMO Systems With Large Antenna Arrays

Xinyu Gao, *Student Member, IEEE*, Linglong Dai, *Senior Member, IEEE*, Shuangfeng Han, *Member, IEEE*,  
Chih-Lin I, *Senior Member, IEEE*, and Robert W. Heath Jr., *Fellow, IEEE*

- SPS partially-connected structure: **Energy efficiency**
- Concept of successive interference cancellation (SIC) was transplanted to design the precoding algorithm



# Improve Spectral Efficiency

## (II) Partially-Connected Mapping

### Existing work

Apr. 2016

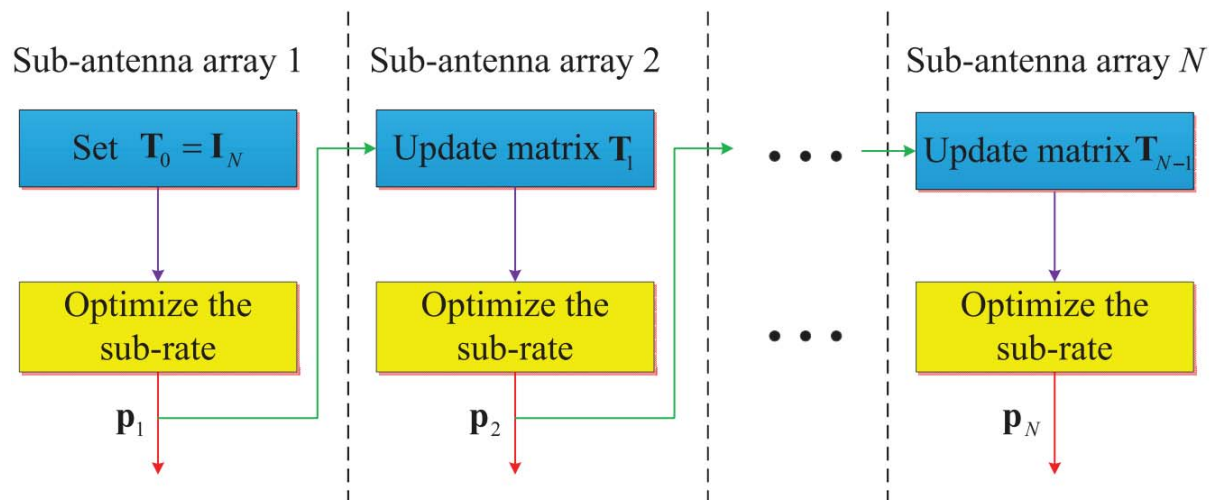


Fig. 2. Diagram of the proposed SIC-based hybrid precoding.

➤ Q: How to directly design hybrid beamforming with the partially-connected mapping?

# Improve Spectral Efficiency

## (II) Partially-Connected Mapping

### ❖ SPS partially-connected

- $\mathcal{A}_x$ : Block diagonal  $\mathbf{F}_{\text{RF}}$  with unit modulus non-zero elements

$$\mathbf{F}_{\text{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\text{RF}}^t} \end{bmatrix} \quad \mathbf{p}_i = \left[ \exp \left( j\theta_{(i-1)\frac{N_t}{N_{\text{RF}}^t} + 1} \right), \cdots, \exp \left( j\theta_i \frac{N_t}{N_{\text{RF}}^t} \right) \right]^T$$

phase shifters connected to the  $i$ -th RF chain

- Problem decoupled for each RF chain

- Closed-form solution for  $\mathbf{F}_{\text{RF}}$

$$\arg \{(\mathbf{F}_{\text{RF}})_{i,l}\} = \arg \left\{ (\mathbf{F}_{\text{opt}})_{i,:} (\mathbf{F}_{\text{BB}})_{l,:}^H \right\}, \quad 1 \leq i \leq N_t, \quad l = \left\lceil i \frac{N_{\text{RF}}^t}{N_t} \right\rceil$$

# Improve Spectral Efficiency

## (II) Partially-Connected Mapping

### ❖ SPS partially-connected (cont.)

#### ➤ Optimization of $\mathbf{F}_{\text{BB}}$

$$\begin{aligned} & \underset{\mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && \|\mathbf{F}_{\text{BB}}\|_F^2 = \frac{N_{\text{RF}}^t N_s}{N_t}. \end{aligned}$$

#### ➤ Reformulate as a non-convex problem

$$\begin{aligned} & \underset{\mathbf{Y} \in \mathbb{H}^n}{\text{minimize}} && \text{Tr}(\mathbf{C}\mathbf{Y}) \\ & \text{subject to} && \begin{cases} \text{Tr}(\mathbf{A}_1\mathbf{Y}) = \frac{N_{\text{RF}}^t N_s}{N_t} \\ \text{Tr}(\mathbf{A}_2\mathbf{Y}) = 1 \\ \mathbf{Y} \succeq 0, \text{rank}(\mathbf{Y}) = 1 \end{cases} \end{aligned}$$

convex

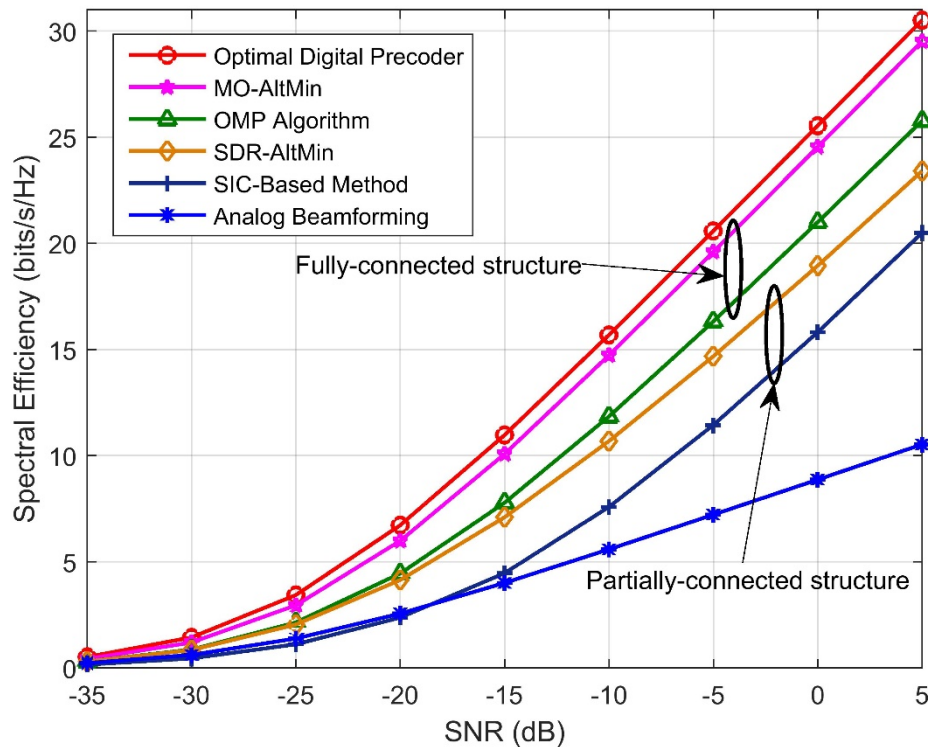
$$\begin{aligned} n &= N_{\text{RF}}^t N_s + 1, \mathbf{y} = [\text{vec}(\mathbf{F}_{\text{BB}}) \quad t]^T, \\ \mathbf{Y} &= \mathbf{y}\mathbf{y}^H, \mathbf{f} = \text{vec}(\mathbf{F}_{\text{opt}}), \\ \mathbf{A}_1 &= \begin{bmatrix} \mathbf{I}_{n-1} & \mathbf{0} \\ \mathbf{0} & 0 \end{bmatrix}, \mathbf{A}_2 = \begin{bmatrix} \mathbf{0}_{n-1} & \mathbf{0} \\ \mathbf{0} & 1 \end{bmatrix}, \\ \mathbf{C} &= \begin{bmatrix} (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}})^H (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}}) & -(\mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}})^H \mathbf{f} \\ -\mathbf{f}^H (\mathbf{I}_{N_s} \otimes \mathbf{F}_{\text{RF}}) & \mathbf{f}^H \mathbf{f} \end{bmatrix}. \end{aligned}$$

#### ➤ Semidefinite relaxation (SDR) is tight for this case so globally optimal solution is obtained [Z.-Q. Luo et al., 2010]

# Improve Spectral Efficiency

## ❖ Simulation results

$$N_t = 144, N_r = 36, N_{\text{RF}}^t = N_{\text{RF}}^r = N_s = 3$$

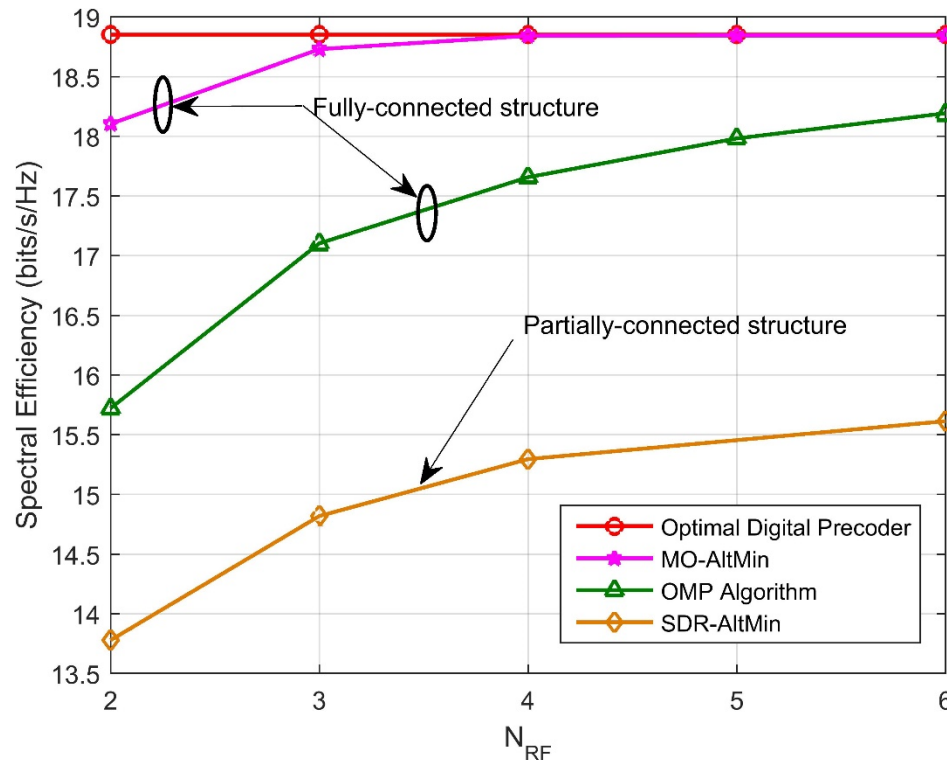


- Effectiveness of the proposed AltMin algorithms
- The fully-connected mapping can easily approach the performance of the fully digital precoding

# Improve Spectral Efficiency

## ❖ Simulation results

$$N_t = 144, N_r = 36, N_{RF}^t = N_{RF}^r = N_{RF}, N_s = 2, \text{SNR} = 0 \text{ dB}$$



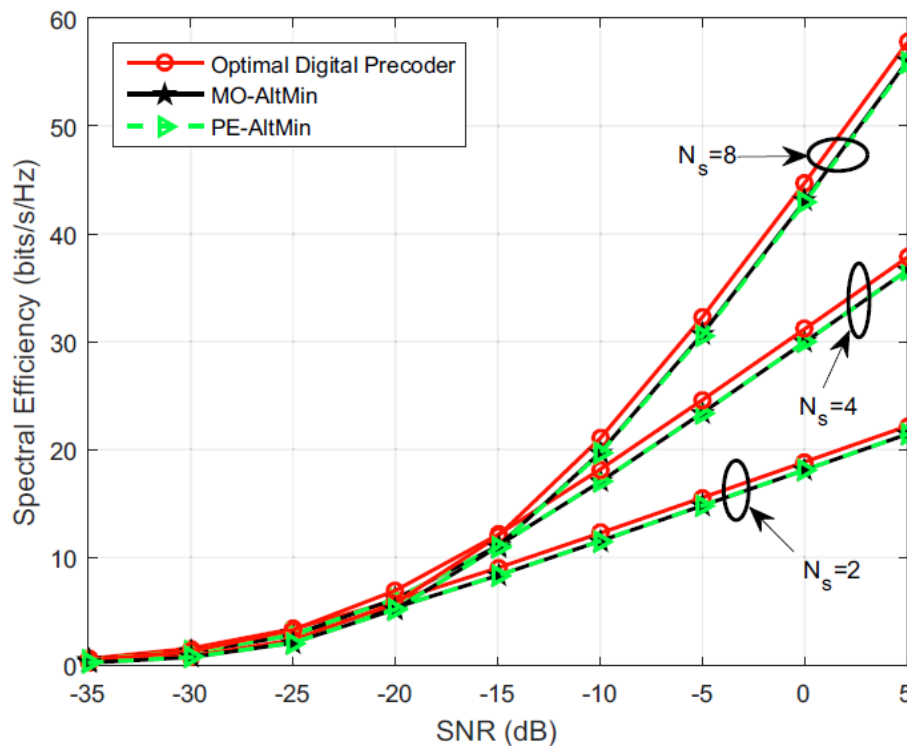
- $\sim N_s$  RF chains are enough for the fully-connected mapping
- Employing fewer PSs, the partially-connected mapping needs more RF chains

**Limitation:** computational efficiency of the MO-AltMin is not good, thus difficult to extend to MU-MC settings

# Improve Spectral Efficiency

## ❖ Simulation results

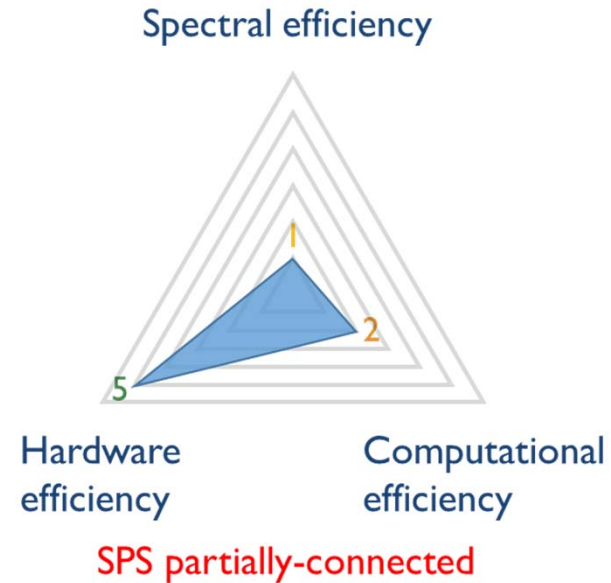
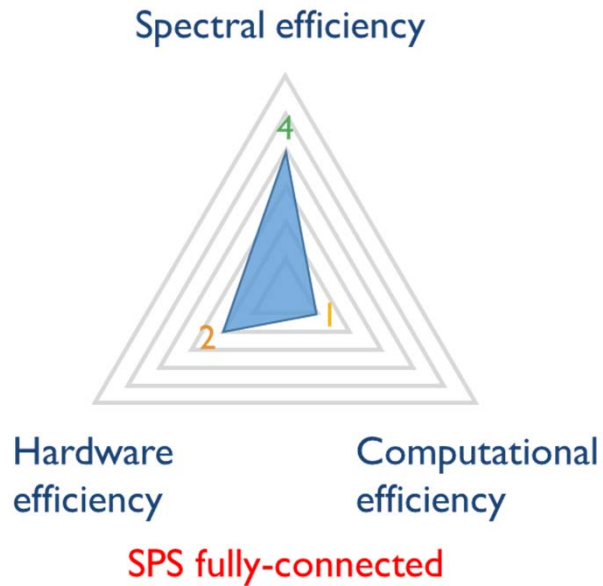
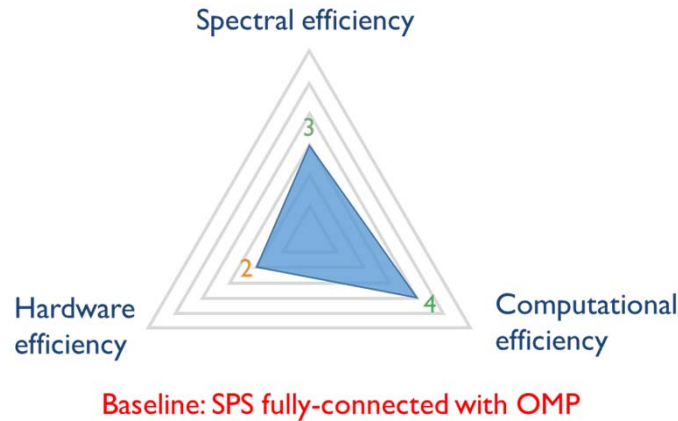
$$N_t = 144, N_r = 36, N_{RF}^t = N_{RF}^r = N_{RF}$$



➤ PE-AltMin algorithm serves as an excellent low-complexity algorithm for hybrid beamforming when  $N_{RF} = N_s$

# Improve Spectral Efficiency

## ❖ Conclusions



## ❖ Other approaches

➤ Apr. 2016

Citation > 225

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 10, NO. 3, APRIL 2016

501

## Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

Foad Sohrabi, *Student Member, IEEE*, and Wei Yu, *Fellow, IEEE*

- Mainly focus on the special case  $N_{\text{RF}}=N_s$
- **Directly maximize the spectral efficiency** with the semi-orthogonal constraint on the digital precoding matrix  $\mathbf{F}_{\text{BB}}$
- Element-wise alternating minimization for the matrix  $\mathbf{F}_{\text{RF}}$



## ❖ Other approaches

➤ Apr. 2016

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 10, NO. 3, APRIL 2016

501

## Hybrid Digital and Analog Beamforming Design for Large-Scale Antenna Arrays

Foad Sohrabi, *Student Member, IEEE*, and Wei Yu, *Fellow, IEEE*

$$\mathbf{F}_1 = \mathbf{H}\mathbf{H}^H$$

$$\mathbf{G}_j = \frac{\gamma^2}{\sigma^2} \mathbf{F}_1 - \frac{\gamma^4}{\sigma^4} \mathbf{F}_1 \bar{\mathbf{V}}_{\text{RF}}^j \mathbf{C}_j^{-1} (\bar{\mathbf{V}}_{\text{RF}}^j)^H \mathbf{F}_1$$

$$\zeta_{ij} = \mathbf{G}_j(i, i) + 2 \operatorname{Re} \left\{ \sum_{m \neq i, n \neq i} \mathbf{V}_{\text{RF}}^*(m, j) \mathbf{G}_j(m, n) \mathbf{V}_{\text{RF}}(n, j) \right\}$$

$$\eta_{ij} = \sum_{\ell \neq i} \mathbf{G}_j(i, \ell) \mathbf{V}_{\text{RF}}(\ell, j)$$

$$\mathbf{F}_{\text{RF}}(i, j) = \begin{cases} \frac{\eta_{ij}}{|\eta_{ij}|} & \eta_{ij} \neq 0, \\ 1 & \eta_{ij} = 0 \end{cases}$$

# Boost Computational Efficiency: Convex Relaxation

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “Alternating minimization for hybrid precoding in multiuser OFDM mmWave Systems,” in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2016. **(Invited Paper)**

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “Partially-connected hybrid precoding in mm-wave systems with dynamic phase shifter networks,” in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Sapporo, Japan, Jul. 2017.

## ❖ Existing works

➤ Jan. 2015

Citation > 73

IEEE TRANSACTIONS ON SIGNAL PROCESSING, VOL. 63, NO. 2, JANUARY 15, 2015

305

## A Hybrid RF/Baseband Precoding Processor Based on Parallel-Index-Selection Matrix-Inversion-Bypass Simultaneous Orthogonal Matching Pursuit for Millimeter Wave MIMO Systems

Yun-Yueh Lee, Ching-Hung Wang, and Yuan-Hao Huang, *Member, IEEE*

$$\begin{array}{l}
 6: \quad \mathbf{F}_{\text{RF}} = \left[ \mathbf{F}_{\text{RF}} | \mathbf{A}_t^{(k)} \right] \\
 7: \quad \mathbf{F}_{\text{BB}} = (\mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{RF}})^{-1} \mathbf{F}_{\text{RF}}^* \mathbf{F}_{\text{opt}}
 \end{array}
 \quad \rightarrow \quad
 \begin{array}{l}
 6: \quad \mathbf{A} = \mathbf{G}_{k, \mathcal{I}_{i-1}} \mathbf{G}_{\mathcal{I}_{i-1}, \mathcal{I}_{i-1}}^{-1} \\
 7: \quad V = 1 / (\mathbf{G}_{k, k} - \mathbf{A} \mathbf{G}_{\mathcal{I}_{i-1}, k}) \\
 8: \quad \mathbf{M} = \mathbf{A} \Psi_0(\mathcal{I}_{i-1}, :) - \Psi_0(k, :) \\
 9: \quad \mathcal{I}_i = [\mathcal{I}_{i-1} | k], \bar{\mathcal{I}}_i = \bar{\mathcal{I}}_{i-1} - \{k\} \\
 10: \quad \mathbf{G}_{\mathcal{I}_i, \mathcal{I}_i}^{-1} = \begin{bmatrix} \mathbf{G}_{\mathcal{I}_{i-1}, \mathcal{I}_{i-1}}^{-1} + V \mathbf{A}^H \mathbf{A} & -V \mathbf{A}^H \\ -V \mathbf{A} & V \end{bmatrix} \\
 11: \quad \mathbf{X}_i = \begin{bmatrix} \mathbf{X}_{i-1} + V \mathbf{A}^H \mathbf{M} \\ -V \mathbf{M} \end{bmatrix} \\
 12: \quad \Psi_i = \Psi_{i-1}(\bar{\mathcal{I}}_i, :) - \mathbf{G}_{\bar{\mathcal{I}}_i, \mathcal{I}_i}^{-1} \begin{bmatrix} V \mathbf{A}^H \mathbf{M} \\ -V \mathbf{M} \end{bmatrix}
 \end{array}$$

## ❖ Existing works

➤ Dec. 2014

Citation > 254

IEEE WIRELESS COMMUNICATIONS LETTERS, VOL. 3, NO. 6, DECEMBER 2014

653

## Low-Complexity Hybrid Precoding in Massive Multiuser MIMO Systems

Le Liang, *Student Member, IEEE*, Wei Xu, *Member, IEEE*, and Xiaodai Dong, *Senior Member, IEEE*

➤ Low-complexity algorithm based on channel phase extraction

$$\mathbf{F}_{\text{RF}} = \exp\{j\angle(\mathbf{H})\}$$

➤ Enables asymptotic performance analysis with Rayleigh fading

➤ Can only deal with **single-antenna** multiuser MIMO and  $N_{\text{RF}}=K$

- ❖ Main approaches to handle the unit modulus constraints
  - Candidate set/codebook based, with unit modulus elements
    - E.g., OMP
  - Manifold optimization – directly tackle unit modulus constraints
    - E.g., MO-AltMin
  - Phase extraction
    - E.g., Liang et al., WCL 14.
  - **Convex relaxation**

# Boost Computational Efficiency

## (I) Fully-Connected Mapping

### ❖ Main difficulty in designing the SPS implementation

- Analog precoder with the **unit modulus constraints**

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

- An intuitive way to boost computational efficiency is to relax this highly non-convex constraint as a convex one

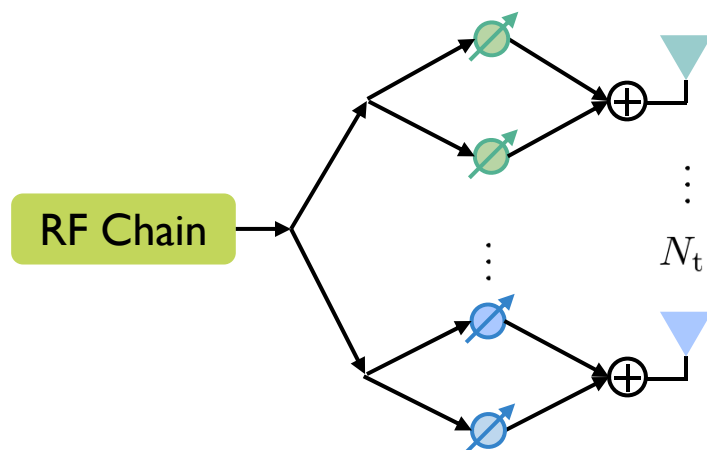
$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && |(\mathbf{F}_{\text{RF}})_{i,j}| \leq \gamma, \forall i, j. \end{aligned}$$

- The value of  $\gamma$  does not affect the hybrid beamformer design
- We shall choose  $\gamma=2$  instead of keeping it as 1. **Why?**

# Boost Computational Efficiency

## ❖ Double phase shifter (DPS) implementation

- The relaxed solution with  $\gamma=2$  can be realized by a hardware implementation



- Unit modulus constraint is eliminated

- Sum of two phase shifters

$$|e^{j\theta_1} + e^{j\theta_2}| \leq 2$$

# Boost Computational Efficiency

## (I) Fully-Connected Mapping

### ❖ Fully-connected mapping

#### ➤ RF-only precoding

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}}{\text{minimize}} \quad \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} \quad |(\mathbf{F}_{\text{RF}})_{i,j}| \leq 2 \end{aligned} \quad \longleftrightarrow \quad \underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{2} \|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + 2\|\mathbf{x}\|_1$$

**LASSO**

#### ➤ Closed-form solution for semi-unitary codebooks $\mathbf{F}_{\text{BB}}\mathbf{F}_{\text{BB}}^H = \mathbf{I}_{N_{\text{RF}}^t}$

$$\mathbf{F}_{\text{RF}}^* = \mathbf{F}_{\text{opt}}\mathbf{F}_{\text{BB}}^H - \exp\{j\angle(\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{BB}}^H)\} \circ (|\mathbf{F}_{\text{opt}}\mathbf{F}_{\text{BB}}^H| - 2)^+.$$

#### ➤ Hybrid precoding

$$\begin{aligned} & \underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} \quad |(\mathbf{F}_{\text{RF}})_{i,j}| \leq 2 \end{aligned} \quad \longrightarrow \quad \text{Matrix factorization}$$

**Redundant**



# Boost Computational Efficiency

## (I) Fully-Connected Mapping

### ❖ Fully-connected mapping (cont.)

#### ➤ Optimality in **single-carrier** systems

$$\mathbf{F}_{\text{opt}} = \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}} \text{ with } \underline{N_{\text{RF}}^t = KN_s} \text{ and } \underline{N_{\text{RF}}^r = N_s} \text{ when } F = 1$$

Minimum number of RF chains

#### ➤ It reduces the required number of RF chains **by half** for achieving the fully digital precoding

#### ➤ **Multi-carrier** systems

$$\underset{\mathbf{F}_{\text{RF}}, \mathbf{F}_{\text{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\text{opt}} - \mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}\|_F^2$$

#### ➤ Low-rank matrix approximation: SVD, **globally optimal solution**

# Boost Computational Efficiency

## (I) Fully-Connected Mapping

### ❖ Fully-connected mapping (cont.)

➤ Q: How to use this relaxed result for SPS implementation?

➤ Optimal solution:

$$\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}} = \mathbf{U}_1\mathbf{S}_1\mathbf{V}_1^H$$

➤ Some clues: The unitary matrix  $\mathbf{U}_1$  fully extracts the information of the column space of  $\mathbf{F}_{\text{RF}}\mathbf{F}_{\text{BB}}$ , whose basis are the orthonormal columns in  $\mathbf{F}_{\text{RF}}$

➤ Phase extraction

$$\mathbf{F}_{\text{RF}} = \exp\{j\angle(\mathbf{U}_1)\}, \quad \mathbf{F}_{\text{BB}} = \mathbf{S}_1\mathbf{V}_1^H$$

unit modulus constraint

Convex relaxation-enabled  
(CR-enabled) SPS

# Boost Computational Efficiency

## (II) Partially-Connected Mapping

### ❖ Partially-connected mapping

#### ➤ Block diagonal structure

$$\mathbf{F}_{\text{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\text{RF}}^t} \end{bmatrix} \quad \mathbf{p}_j = \left[ a_{(j-1)\frac{N_t}{N_{\text{RF}}^t} + 1}, \dots, a_{j\frac{N_t}{N_{\text{RF}}^t}} \right]^T$$

#### ➤ Decoupled for each RF chain

$$\mathcal{P}_j : \underset{\{a_i\}, \mathbf{x}_j}{\text{minimize}} \sum_{i \in \mathcal{F}_j} \|\mathbf{y}_i - a_i \mathbf{x}_j\|_2^2,$$

$$\mathcal{F}_j = \left\{ i \in \mathbb{Z} \mid (j-1)\frac{N_t}{N_{\text{RF}}^t} + 1 \leq i \leq j\frac{N_t}{N_{\text{RF}}^t} \right\}, \mathbf{y}_i = \mathbf{F}_{\text{opt}}^T(i, :), \text{ and } \mathbf{x}_j = \mathbf{F}_{\text{BB}}^T(j, :)$$


#### ➤ Eigenvalue problem

$$\mathbf{x}_j^* = \lambda_1 \left( \sum_{i \in \mathcal{F}_j} \mathbf{y}_i \mathbf{y}_i^H \right), \quad a_i^* = \frac{\mathbf{x}_j^H \mathbf{y}_i}{\|\mathbf{x}_j\|_2^2}$$

# Boost Computational Efficiency

## (II) Partially-Connected Mapping

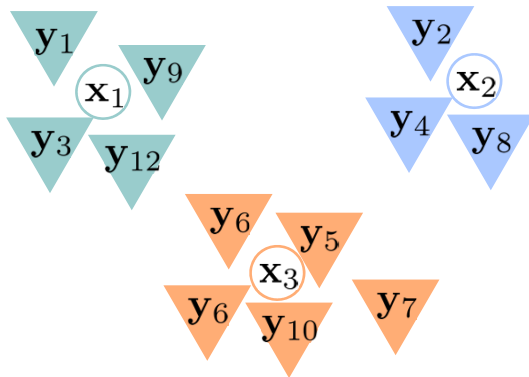
### ❖ DPS partially-connected mapping (cont.)

➤ Not much performance gain obtained by simply adopting the DPS implementation 

➤ **Dynamic mapping:**

**Adaptively separate all  $N_t$  antennas into  $N_{\text{RF}}$  groups**

$$\text{maximize}_{\{\mathcal{D}_j\}_{j=1}^{N_{\text{RF}}^t}} \sum_{j=1}^{N_{\text{RF}}^t} \lambda_1 \left( \sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right)$$



➤ **Modified K-means algorithm**

■ **Centroid:**  $\mathbf{x}_j^* = \lambda_1 \left( \sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right)$

■ **Clustering:**  $j^* = \arg \max_j |\mathbf{y}_i^H \mathbf{x}_j|^2$

➤ **Convergence guarantee**

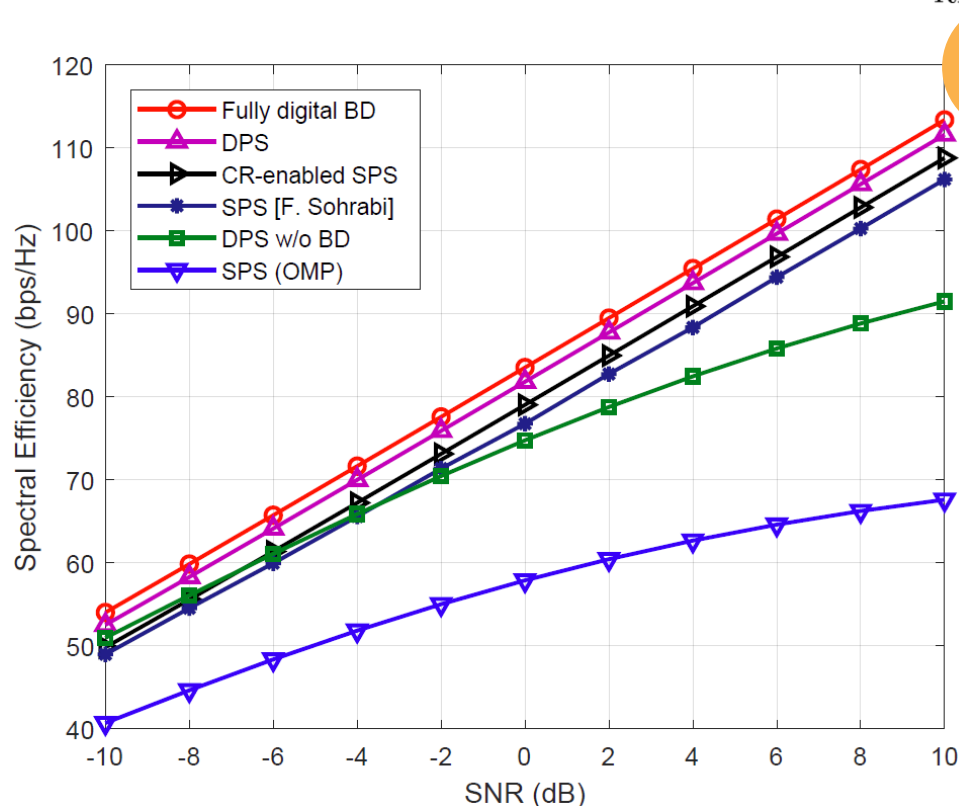
## ❖ MU-MC systems: Inter-user interference

- Approximating the fully digital precoder leads to **near-optimal performance** in single-user single-carrier, single-user multicarrier, and multiuser single-carrier mm-wave MIMO systems
- Inter-user interference will be more prominent in multiuser multicarrier systems as **the analog precoder is shared by a large number of subcarriers**
  - **Additional care is needed**
- Cascade an additional block diagonalization (BD) precoder
  - Effective channel:  $\hat{\mathbf{H}}_{k,f} = \mathbf{W}_{\text{BB},k,f}^H \mathbf{W}_{\text{RF},k}^H \mathbf{H}_{k,f} \mathbf{F}_{\text{RF}} \mathbf{F}_{\text{BB},f}$
  - BD:  $\hat{\mathbf{H}}_{j,f} \mathbf{F}_{\text{BD},k,f} = \mathbf{0}, \quad k \neq j$

# Boost Computational Efficiency

## ❖ Simulation results (Fully-connected)

$N_t = 256$ ,  $N_r = 16$ ,  $K = 3$ ,  $F = 128$ ,  $N_s = 3$ ,  $N_{\text{RF}}^t = 9$ , and  $N_{\text{RF}}^r = 3$



➤ Achieve near-optimal spectral efficiency and optimal multiplexing gain with low-complexity algorithms

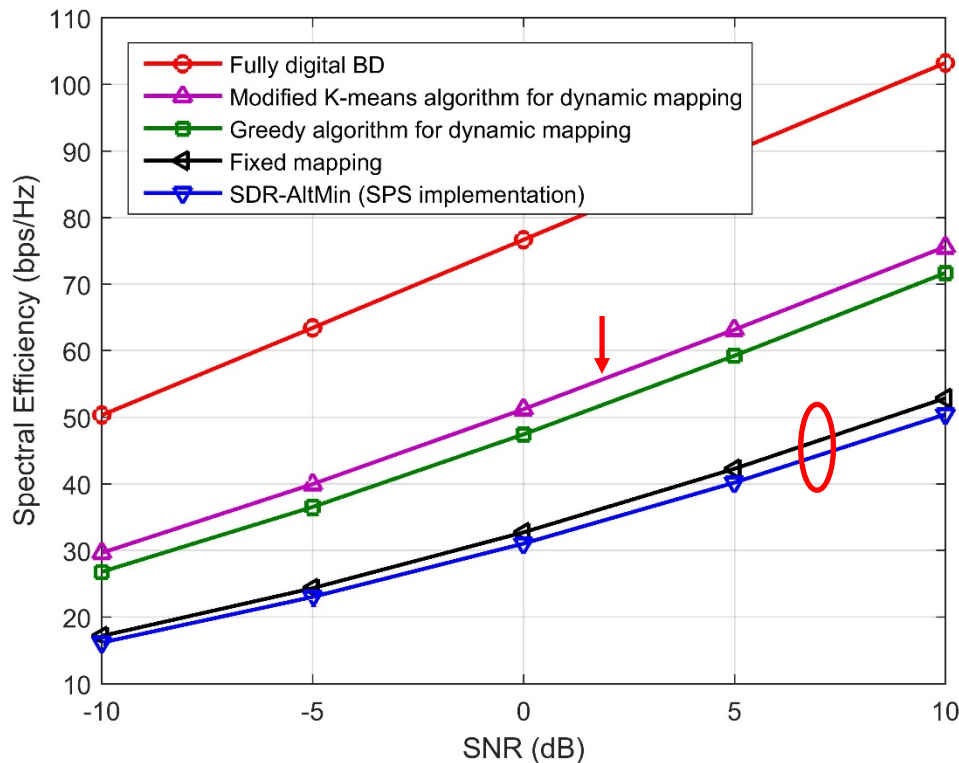
➤ Effectiveness of the proposed CR-enabled SPS method

[Ref] F. Sahrabi and W. Yu, "Hybrid Analog and Digital Beamforming for mmWave OFDM Large-Scale Antenna Arrays," *IEEE J. Sel. Areas Commun.*, vol. 35, no. 7, pp. 1432-1443, July 2017.

# Boost Computational Efficiency

## ❖ Simulation results (Partially-connected)

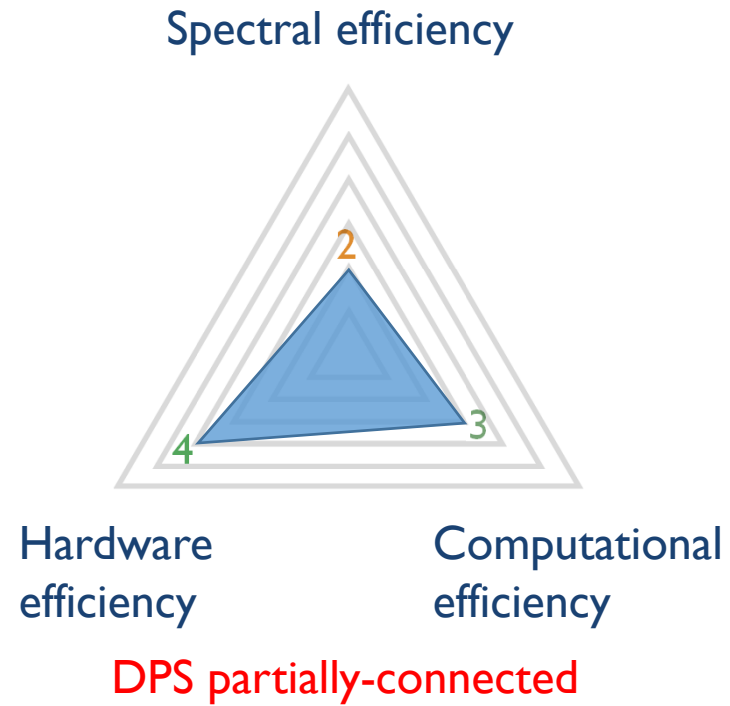
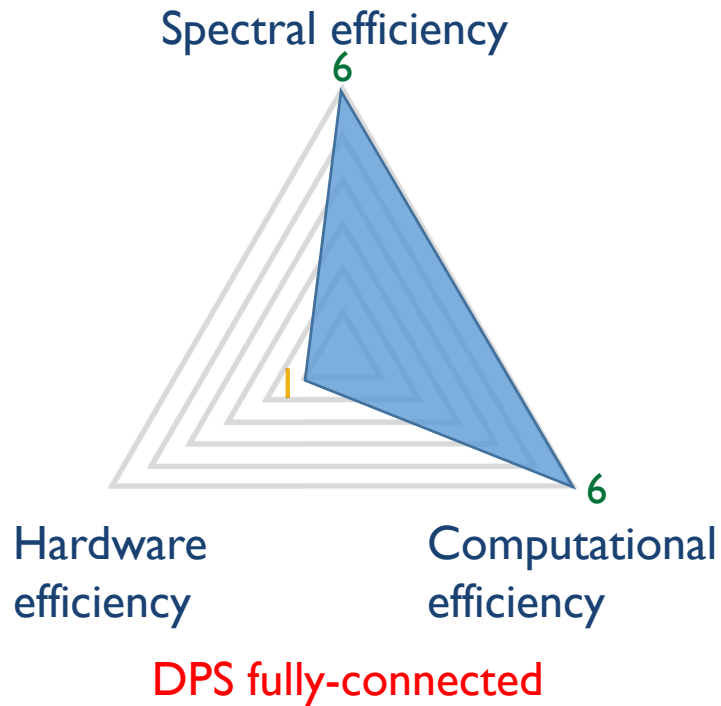
$$N_t = 256, N_r = 16, K = 4, F = 128, N_s = 2 \quad N_{RF}^t = KN_s, \text{ and } N_{RF}^r = N_s$$



- Simply doubling PSs in the partially-connected mapping is far from satisfactory
- Superiority of the modified K-means algorithm with lower computational complexity than the greedy algorithm

# Boost Computational Efficiency

## ❖ Conclusions





# Boost Computational Efficiency

## ❖ Discussions

### ➤ Comparison of computational complexity

Implementation	Structure	Design approach	Hardware complexity (No. of phase shifters)	Computational complexity	Performance
SPS	Fully-connected	MO-AltMin	$N_{\text{RF}}^t N_t$	Extremely high	✓✓✓
	Partially-connected	SDR-AltMin	$N_t$	High	✓
DPS	Fully-connected	Matrix decomposition	$2N_{\text{RF}}^t(N_t - N_{\text{RF}}^t)$	$\mathcal{O}(N_{\text{RF}}^t{}^2 N_t F)$	✓✓✓✓
	Partially-connected	Modified K-means	$2N_t$	$\mathcal{O}(N N_{\text{RF}}^t{}^2 N_t F)$	✓✓

### ➤ The proposed DPS implementation enables low complexity design for hybrid beamforming

# Boost Computational Efficiency

## ❖ Discussions

- The number of RF chains has been reduced to the minimum

$$N_{\text{RF}}^t = KN_s$$

- A large number of high-precision phase shifters are still needed

	Fully-connected	Partially-connected
SPS	$N_t N_{\text{RF}}$	$N_t$
DPS	$2N_t N_{\text{RF}}$	$2N_t$

- Need to adapt the phases to channel states

- ❖ Practical phase shifters are typically with coarsely **quantized** phases

**How to reduce # phase shifters?**

# Fight for Hardware Efficiency: How Many Phase Shifters Are Needed?

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “Hybrid precoding in millimeter wave systems: How many phase shifters are needed?” in *Proc. IEEE Global Commun. Conf. (Globecom)*, Singapore, Dec. 2017. **(Best Paper Award)**

[Ref] X. Yu, J. Zhang, and K. B. Letaief, “A hardware-efficient analog network structure for hybrid precoding in millimeter wave systems,” *IEEE J. Sel. Topics Signal Process., Special Issue on Hybrid Analog-Digital Signal Processing for Hardware-Efficient Large Scale Antenna Arrays*, vol. 12, no. 2, pp. 282-297, May 2018.

# Fight for Hardware Efficiency

## ❖ Commonly-used hardware in hybrid beamforming

Switch ~ binary



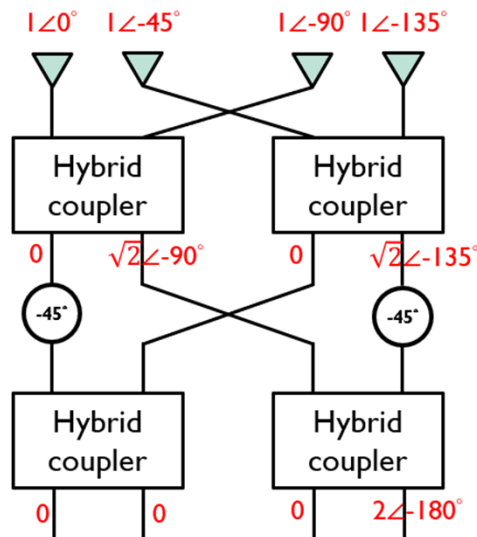
Phase shifter ~ unit modulus



Adaptive

Quantized with fixed phases

Butler matrix ~ FFT matrix



Generate **fixed** phase difference between antenna elements

$$\mathbf{B} = \mathbf{T}\mathbf{F}\mathbf{T}$$

$$\mathbf{F} = \text{FFT}(N_t) \quad \mathbf{T} = \text{diag} \left[ e^{j0}, e^{-j\frac{\pi}{N_t}}, \dots, e^{-j\left(\pi + \frac{\pi}{N_t}\right)} \right]$$

## ❖ Different implementations

TABLE I  
COMPARISONS OF HARDWARE COMPONENTS IN THE ANALOG NETWORK FOR DIFFERENT HYBRID PRECODER STRUCTURES

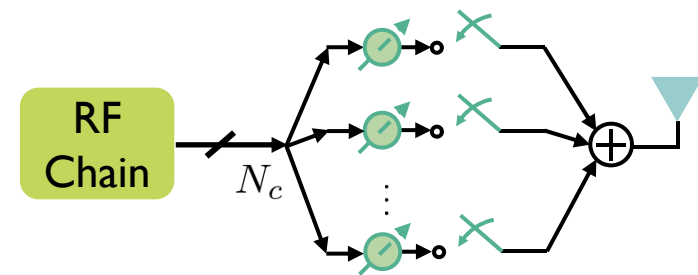
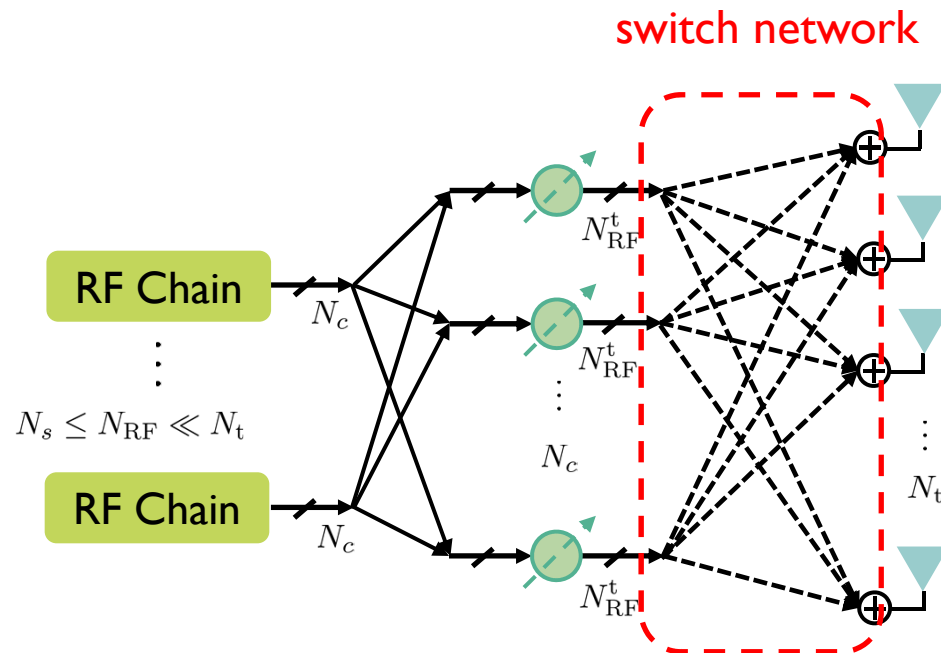
		Phase shifter			Other hardware components		
		Number $N_{PS}$	Type	Power $P_{PS}$	Hardware	Number $N_{OC}$	Power $P_{OC}$
SPS	Fully-connected	$N_{RF}^t N_t$	Adaptive	50 mW	N/A	N/A	N/A
	Partially-connected	$N_t$					
SPS with Butler matrices	Fully-connected	$\frac{N_{RF}^t N_t}{2} (\log_2 N_t - 1)$	Fixed	20 mW	Coupler	$\frac{N_{RF}^t N_t}{2} \log_2 N_t$	10 mW
	Partially-connected	$\frac{N_t}{2} \left( \log_2 \frac{N_t}{N_{RF}^t} - 1 \right)$					
DPS	Fully-connected	$2N_{RF}^t N_t$	Adaptive	50 mW	N/A	N/A	N/A
	Partially-connected	$2N_t$					
FPS	Fully-connected	$N_c \ll N_t$	Multi-channel	20 mW	Switch	$N_c N_{RF}^t N_t$	5 mW
	Group-connected		Fixed				

➤ How to reduce the overall hardware complexity while maintaining good performance?

# Fight for Hardware Efficiency

## (I) Fixed phase shifter implementation

### ❖ Fixed phase shifter (FPS) implementation



Q: How to design these adaptive switches?

➤  $N_c$  multi-channel **fixed PSs** [Z. Feng et al., 2014]

# Fight for Hardware Efficiency

## (I) Fixed phase shifter implementation

### ❖ Problem formulation

➤  $\mathcal{A}_x: \mathbf{F}_{\text{RF}} = \mathbf{S}\mathbf{C}$

➤ FPS matrix  $\mathbf{C} = \text{diag}(\overbrace{\mathbf{c}, \mathbf{c}, \dots, \mathbf{c}}^{N_{\text{RF}}^t}), \quad \mathbf{c} = \frac{1}{\sqrt{N_c}} [e^{j\theta_1}, e^{j\theta_2}, \dots, e^{j\theta_{N_c}}]^T$

➤ Binary switch matrix  $\mathbf{S} \in \{0, 1\}^{N_t \times N_c N_{\text{RF}}^t}$

Phases are fixed

$$\begin{aligned} & \underset{\mathbf{S}, \mathbf{F}_{\text{BB}}}{\text{minimize}} && \|\mathbf{F}_{\text{opt}} - \mathbf{S}\mathbf{C}\mathbf{F}_{\text{BB}}\|_F^2 \\ & \text{subject to} && \mathbf{S} \in \{0, 1\}^{N_t \times N_c N_{\text{RF}}^t} \end{aligned}$$

NP-hard

### ❖ An objective upper bound enables a low-complexity algorithm

➤ Enforce a semi-orthogonal constraint on  $\mathbf{F}_{\text{BB}}$  [X.Yu et al., 2016]

$$\mathbf{F}_{\text{BB}}^H \mathbf{F}_{\text{BB}} = \alpha^2 \mathbf{F}_{\text{DD}}^H \mathbf{F}_{\text{DD}} = \alpha^2 \mathbf{I}_{KN_s}$$

$$\|\mathbf{F}_{\text{opt}} - \mathbf{S}\mathbf{C}\mathbf{F}_{\text{BB}}\|_F^2 \leq \|\mathbf{F}_{\text{opt}}\|_F^2 - 2\alpha \Re \text{Tr}(\mathbf{F}_{\text{DD}} \mathbf{F}_{\text{opt}}^H \mathbf{S}\mathbf{C}) + \alpha^2 \|\mathbf{S}\|_F^2$$

# Fight for Hardware Efficiency

## (I) Fixed phase shifter implementation

### ❖ Alternating minimization

#### ➤ Digital precoder

$$\begin{aligned} & \underset{\mathbf{F}_{DD}}{\text{maximize}} && \Re \text{Tr} (\mathbf{F}_{DD} \mathbf{F}_{\text{opt}}^H \mathbf{S} \mathbf{C}) \\ & \text{subject to} && \mathbf{F}_{DD}^H \mathbf{F}_{DD} = \mathbf{I}_{KN_s} \end{aligned}$$

#### ➤ Semi-orthogonal Procrustes solution $\mathbf{F}_{DD} = \mathbf{V}_1 \mathbf{U}^H$

$$\alpha \mathbf{F}_{\text{opt}}^H \mathbf{S} \mathbf{C} = \mathbf{U} \Sigma \mathbf{V}_1^H$$

#### ➤ Switch matrix optimization

$$\begin{aligned} & \underset{\alpha, \mathbf{S}}{\text{minimize}} && \left\| \Re (\mathbf{F}_{\text{opt}} \mathbf{F}_{DD}^H \mathbf{C}^H) - \alpha \mathbf{S} \right\|_F^2 \\ & \text{subject to} && \mathbf{S} \in \{0, 1\}^{N_t \times N_c N_{\text{RF}}^t} \end{aligned}$$

#### ➤ Once $\alpha$ is optimized, the optimal $\mathbf{S}$ is determined correspondingly

$$\mathbf{S}^* = \begin{cases} \mathbf{1} \left\{ \Re (\mathbf{F}_{\text{opt}} \mathbf{F}_{DD}^H \mathbf{C}^H) > \frac{\alpha}{2} \mathbf{1}_{N_t \times N_c N_{\text{RF}}^t} \right\} & \alpha > 0 \\ \mathbf{1} \left\{ \Re (\mathbf{F}_{\text{opt}} \mathbf{F}_{DD}^H \mathbf{C}^H) < \frac{\alpha}{2} \mathbf{1}_{N_t \times N_c N_{\text{RF}}^t} \right\} & \alpha < 0 \end{cases}$$



# Fight for Hardware Efficiency

## (I) Fixed phase shifter implementation

### ❖ Alternating minimization (cont.)

#### ➤ Optimization of $\alpha$

$$\alpha^* = \arg \min_{\{\tilde{x}_i, \bar{x}_i\}_{i=1}^n} \{f(\tilde{x}_i), f(\bar{x}_i)\}$$

$$\begin{aligned} \tilde{\mathbf{x}} &= \text{vec}(\Re(\mathbf{F}_{\text{opt}} \mathbf{F}_{\text{DD}}^H \mathbf{C}^H)) \\ \tilde{\mathbf{x}} &\in \mathbb{R}^n, \quad n = N_t N_{\text{RF}}^t N_c \end{aligned} \quad \bar{x}_i \triangleq \begin{cases} \frac{\sum_{j=1}^i \tilde{x}_j}{i} & \alpha < 0 \text{ and } \frac{\sum_{j=1}^i \tilde{x}_j}{i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ \frac{\sum_{j=i+1}^n \tilde{x}_j}{n-i} & \alpha > 0 \text{ and } \frac{\sum_{j=i+1}^n \tilde{x}_j}{n-i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ +\infty & \text{otherwise} \end{cases}$$

#### ➤ Search dimension: $|\mathcal{X}| = 2N_t N_{\text{RF}}^t N_c$



#### ➤ **Acceleration:** Optimal point can only be obtained at $\bar{x}_i$

$$\alpha^* = \arg \min_{\bar{x}_i} f(\bar{x}_i)$$

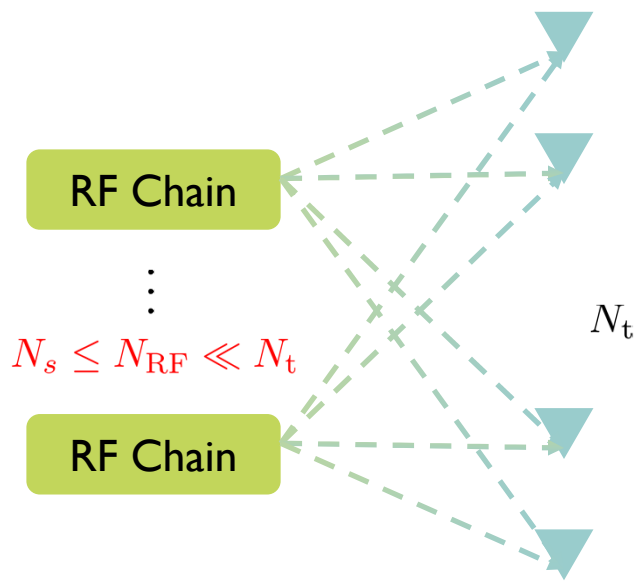
#### ➤ Search dimension $\ll 2N_t N_{\text{RF}}^t N_c$

#### ➤ Convergence guarantee

# Fight for Hardware Efficiency

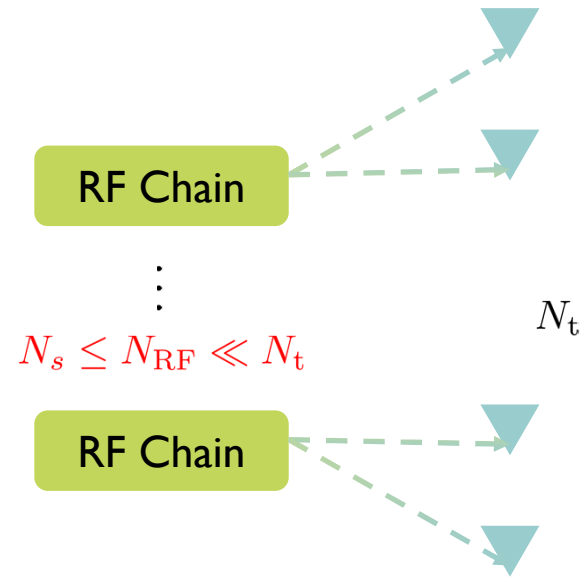
## (II) Flexible hardware-performance tradeoff

❖ Two common mapping strategies



Fully-connected

Performance



Partially-connected

Hardware efficiency

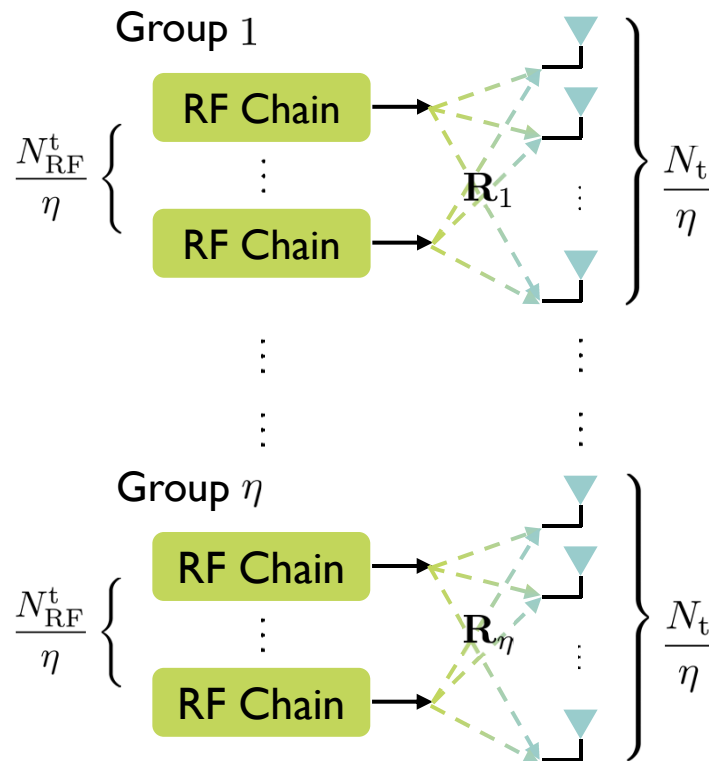


# Fight for Hardware Efficiency

## (II) Flexible hardware-performance tradeoff

### ❖ A mapping strategy for flexible hardware-performance tradeoff

#### ➤ Group-connected mapping



Save hardware by  $\eta$  times

$$\mathbf{F}_{\text{RF}} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_\eta \end{bmatrix}$$

➤  $\eta = 1$ : Fully-connected

➤  $\eta = N_{\text{RF}}^t$ : Partially-connected

$$\text{minimize}_{\mathbf{R}_i, \mathbf{B}_i} \quad \|\mathbf{F}_i - \mathbf{R}_i \mathbf{B}_i\|_F^2$$

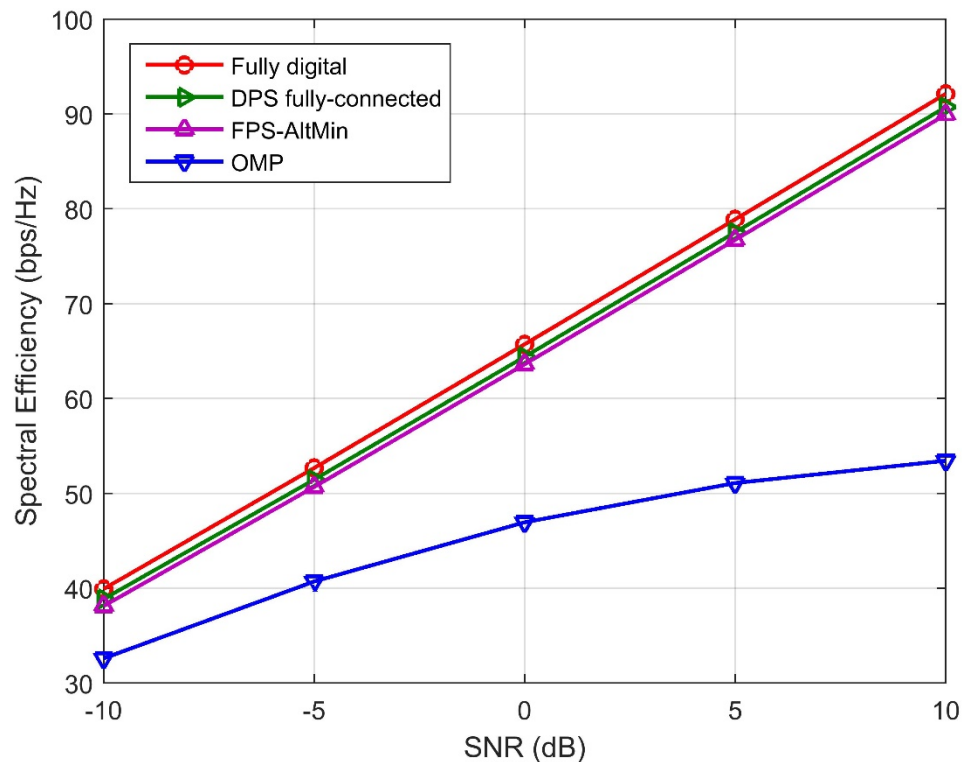
$$\text{subject to} \quad \mathbf{R}_i \in \mathcal{A}_i$$

Directly migrate the design for the fully-connected mapping

# Fight for Hardware Efficiency

## ❖ Simulation results: MU-MC systems

$N_t = 144$ ,  $N_r = 16$ ,  $K = 4$ ,  $F = 128$ ,  $N_s = 2$ ,  $N_{\text{RF}}^t = 8$ , and  $N_{\text{RF}}^r = 2$

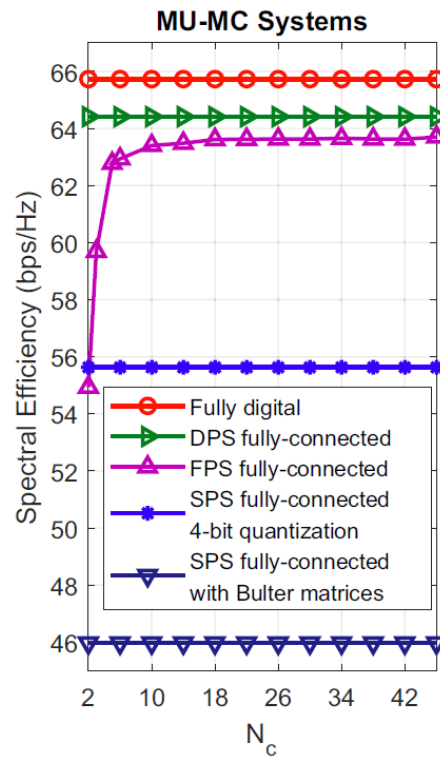
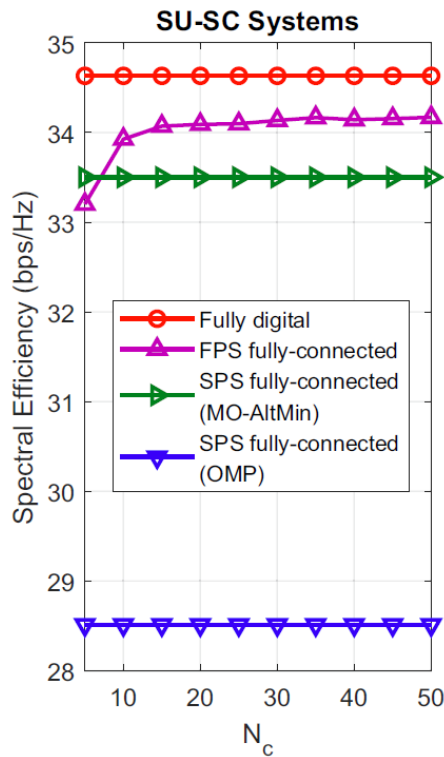


- Slightly inferior to the DPS fully-connected mapping with much fewer PSs
- Significant improvement over the OMP algorithm

# Fight for Hardware Efficiency

## ❖ Simulation results: How many PSs are needed?

$$N_t = 256, N_r = 16, K = 4, F = 128, N_s = 2, N_{RF}^t = 8, \text{ and } N_{RF}^r = 2$$



➤ Only ~10 fixed phase shifters are sufficient!

➤ 200 times reduction compared with the DPS implementation

# Fight for Hardware Efficiency

## ❖ Simulation results: How much power can be saved?

$$N_t = 256, N_r = 16, K = 4, F = 128, N_s = 2, N_{\text{RF}}^t = 8, \text{ and } N_{\text{RF}}^r = 2$$

TABLE II

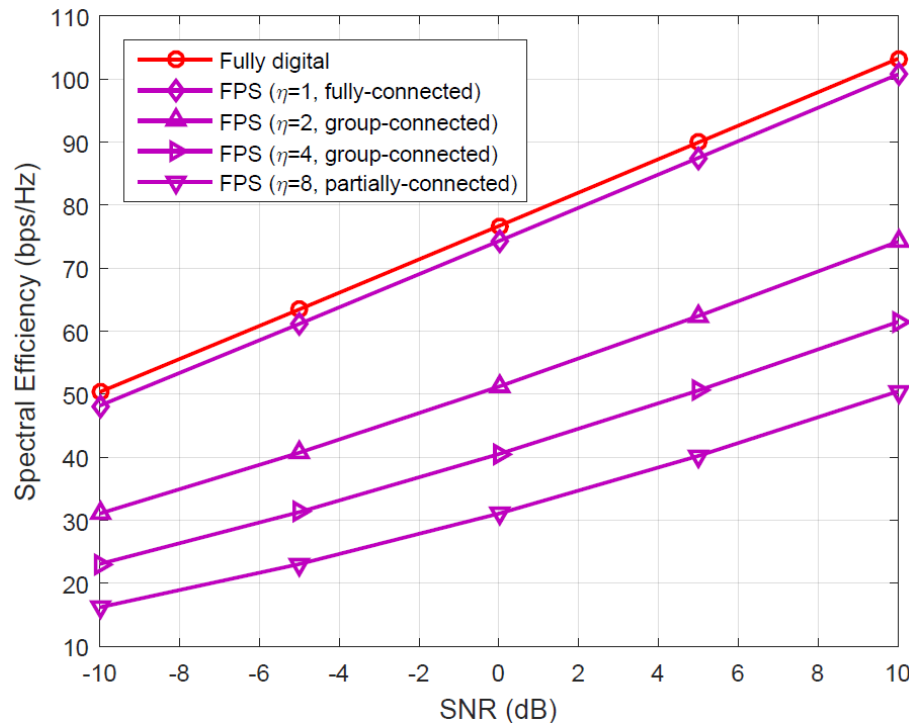
POWER CONSUMPTION OF THE ANALOG NETWORK FOR DIFFERENT HYBRID PRECODER STRUCTURES IN MU-MC SYSTEMS

	Phase shifter		Other hardware		Total power <sup>‡</sup>
	Number $N_{\text{PS}}$	Type	Hardware	Number $N_{\text{OC}}$	$P_{\text{total}}$
<b>DPS fully-connected</b>	2304	Adaptive	N/A	N/A	115.2 W
<b>FPS fully-connected</b>	10	Fixed <sup>§</sup>	Switch	11520	59.2 W
<b>SPS fully-connected 4-bit quantization</b>	1152	Adaptive	N/A	N/A	57.6 W
<b>FPS fully-connected</b>	2	Fixed	Switch	2304	11.84 W
<b>SPS fully-connected with Butler matrices</b>	3456	Fixed	Coupler	4032	109.44 W

# Fight for Hardware Efficiency

## ❖ Simulation results

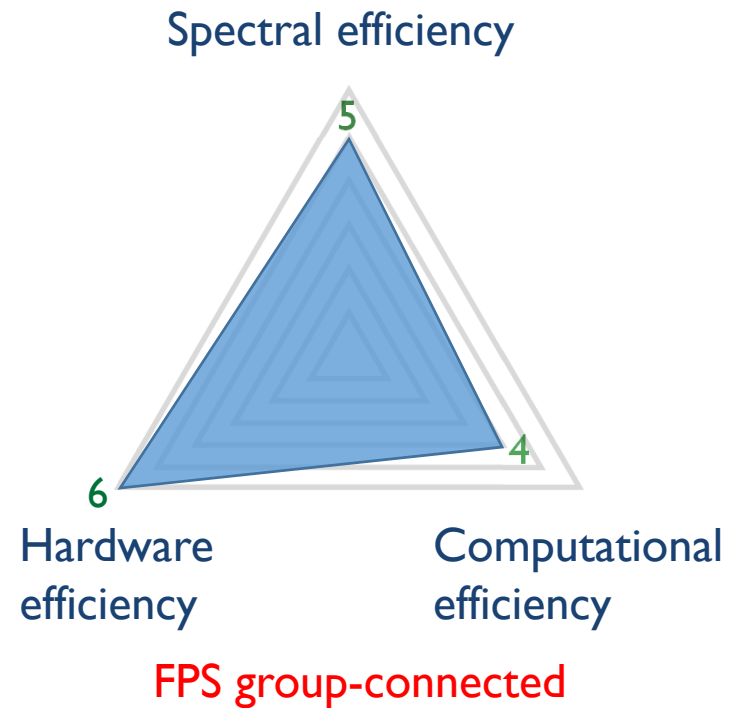
$N_t = 256$ ,  $N_r = 16$ ,  $K = 4$ ,  $F = 128$ ,  $N_s = 2$ ,  $N_{\text{RF}}^t = 8$ , and  $N_{\text{RF}}^r = 2$



A flexible approach to balance the achievable performance and hardware efficiency

# Fight for Hardware Efficiency

## ❖ Conclusions





# Conclusions

## ❖ Questions answered

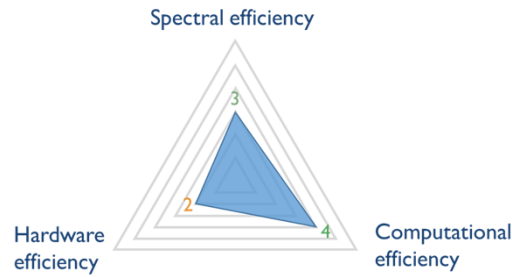
- **Q1:** Can hybrid precoder provide performance close to the fully digital one? **YES**
- **Q2:** How many RF chains are needed?  $KN_s$
- **Q3:** How many phase shifters are needed? **~10 FPSs**
- **Q4:** How to connect the RF chains and antennas? **Group-connected**
- **Q5:** How to efficiently design hybrid precoding algorithms?

Alternating minimization provides the basic principle

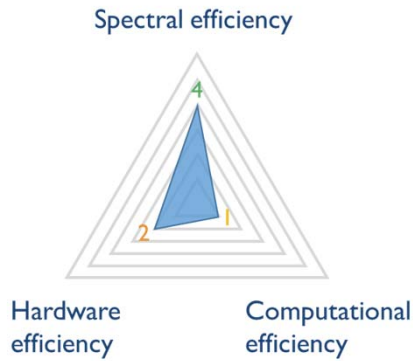
Manifold optimization provides good benchmark

Convex relaxation enables low-complexity algorithms

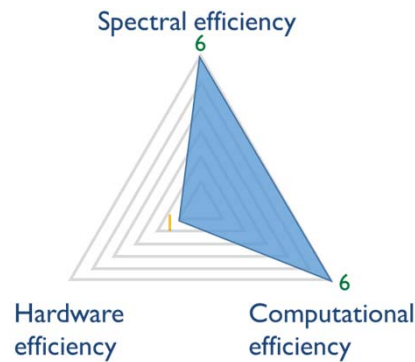
# Conclusions



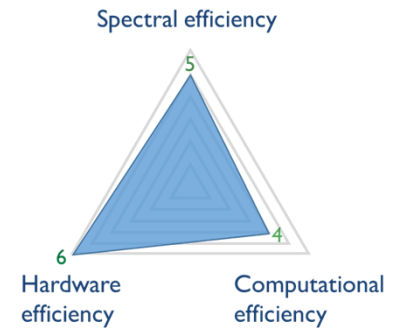
Baseline: SPS fully-connected with OMP



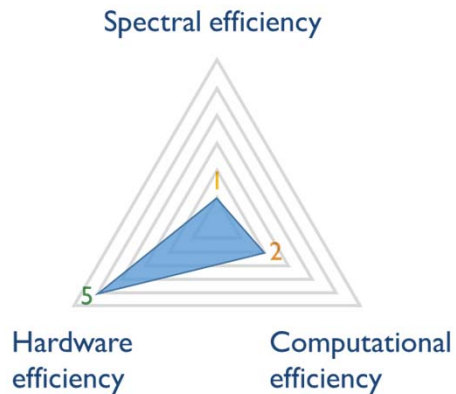
SPS fully-connected



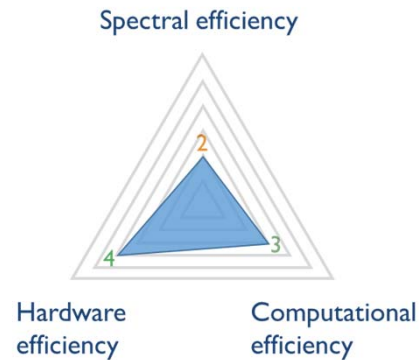
DPS fully-connected



FPS group-connected

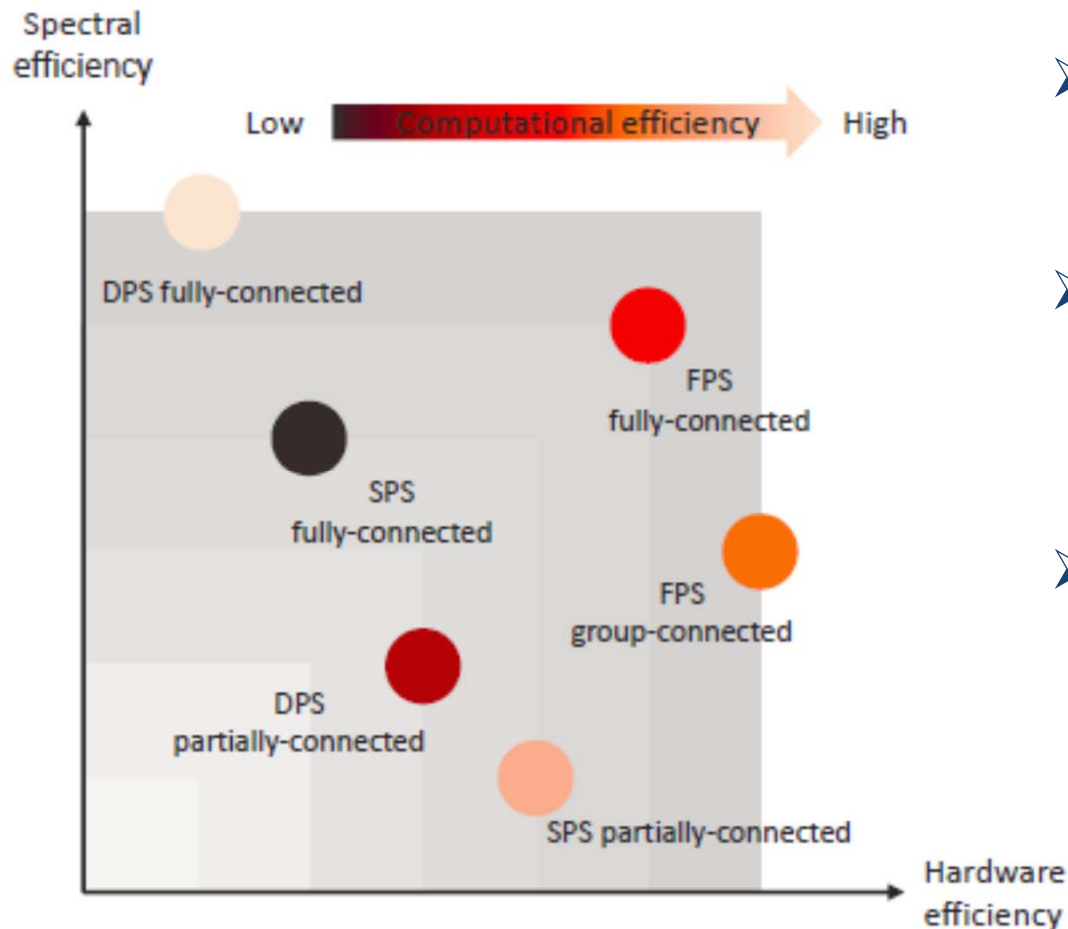


SPS partially-connected



DPS partially-connected

## ❖ Comparisons between different hybrid precoder structures



- SPS: May not be a good choice
- DPS: An excellent candidate for low-complexity algorithms
- FPS: A trade-off between the hardware and computational complexity, with satisfactory performance

# Potential research directions

## ➤ Joint design with CSI acquisition and uncertainty

IEEE JOURNAL OF SELECTED TOPICS IN SIGNAL PROCESSING, VOL. 8, NO. 5, OCTOBER 2014

831

### Channel Estimation and Hybrid Precoding for Millimeter Wave Cellular Systems

Ahmed Alkhateeb, *Student Member, IEEE*, Omar El Ayach, *Member, IEEE*, Geert Leus, and Robert W. Heath, Jr., *Fellow, IEEE*

IEEE COMMUNICATIONS LETTERS, VOL. 20, NO. 6, JUNE 2016

Beam design for the training stage with the hybrid structures

### Channel Estimation for Millimeter-Wave Massive MIMO With Hybrid Precoding Over Frequency-Selective Fading Channels

Zhen Gao, Chen Hu, Linglong Dai, and Zhaocheng Wang

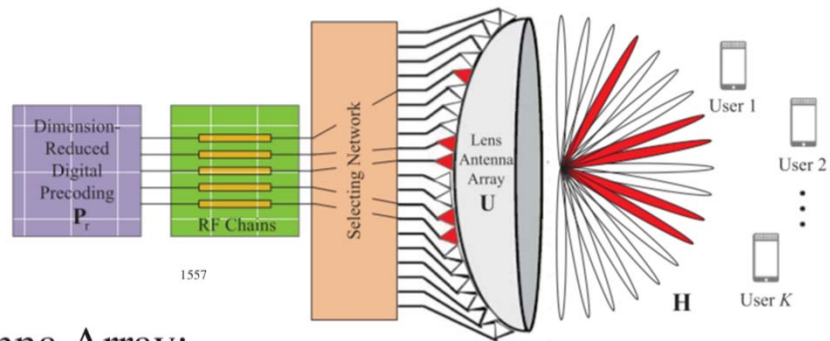
Hybrid precoding with partial CSI or covariance info. only

### Hybrid Precoding for Millimeter Wave Cellular Systems with Partial Channel Knowledge

Ahmed Alkhateeb<sup>1</sup>, Omar El Ayach<sup>1</sup>, Geert Leus<sup>2</sup>, and Robert W. Heath Jr.<sup>1</sup>  
<sup>1</sup> The University of Texas at Austin, Email: {alkhateeb, oelayach, rheath}@utexas.edu  
<sup>2</sup> Delft University of Technology, Email: g.j.t.leus@tudelft.nl

# Potential research directions

## ➤ Comparison between different antenna configurations



IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 64, NO. 4, APRIL 2016

### Millimeter Wave MIMO With Lens Antenna Array: A New Path Division Multiplexing Paradigm

Yong Zeng, *Member, IEEE*, and Rui Zhang, *Senior Member, IEEE*

10

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 16, NO. 9, SEPTEMBER 2017

### Reliable Beamspace Channel Estimation for Millimeter-Wave Massive MIMO Systems with Lens Antenna Array

Xinyu Gao, *Student Member, IEEE*, Linglong Dai, *Senior Member, IEEE*, Shuangfeng Han, *Member, IEEE*,  
Chih-Lin I, *Senior Member, IEEE*, and Xiaodong Wang, *Fellow, IEEE*

Hybrid beamforming and  
channel estimation with lens  
antenna arrays

# Potential research directions

## ➤ Hybrid beamforming for THz communications

IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS, VOL. 14, NO. 6, JUNE 2015

3097

### Indoor Terahertz Communications: How Many Antenna Arrays Are Needed?

Cen Lin and Geoffrey Ye Li, *Fellow, IEEE*

How to use antennas efficiently?



### Antenna Subarray Partitioning with Interference Cancellation for Multi-User Indoor Terahertz Communications

Cen Lin and Geoffrey Ye Li  
School of ECE, Georgia Institute of Technology, Atlanta, GA 30332, USA  
Email: linc@gatech.edu, liye@ece.gatech.edu

# Potential research directions

## ➤ Performance evaluation

IEEE WIRELESS COMMUNICATIONS LETTERS, VOL. 3, NO. 6, DECEMBER 2014

653

### Low-Complexity Hybrid Precoding in Massive Multiuser MIMO Systems

Le Liang, *Student Member, IEEE*, Wei Xu, *Member, IEEE*, and Xiaodai Dong, *Senior Member, IEEE*

### Performance characterization of hybrid precoding

52

IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 64, NO. 5, MAY 2016

### A Comparison of MIMO Techniques in Downlink Millimeter Wave Cellular Networks With Hybrid Beamforming

Mandar N. Kulkarni, *Student Member, IEEE*, Amitava Ghosh, *Fellow, IEEE*, and Jeffrey G. Andrews, *Fellow, IEEE*

Comparison between  
MU-MIMO and single  
user spatial multiplexing



# Potential research directions

- Further reduction in computational complexity

## Machine Learning Inspired Energy-Efficient Hybrid Precoding for MmWave Massive MIMO Systems

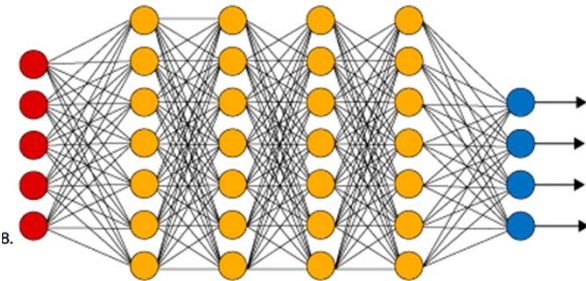
Xinyu Gao\*, Linglong Dai\*, Ying Sun\*, Shuangfeng Han<sup>†</sup>, and Chih-Lin I<sup>†</sup>

\*Tsinghua National Laboratory for Information Science and Technology (TNList),  
Department of Electronic Engineering, Tsinghua University, Beijing, China

<sup>†</sup>Green Communication Research Center, China Mobile Research Institute, Beijing 100053, China

Received April 26, 2018, accepted May 29, 2018, date of publication June 25, 2018, date of current version July 25, 2018.

Digital Object Identifier 10.1109/ACCESS.2018.2850226



## Deep Learning Coordinated Beamforming for Highly-Mobile Millimeter Wave Systems

**AHMED ALKHATEEB<sup>1</sup>**, **SAM ALEX<sup>2</sup>**, **PAUL VARKEY<sup>2</sup>**, **YING LI<sup>2</sup>**,  
**QI QU<sup>2</sup>**, AND **DJORDJE TUJKOVIC<sup>2</sup>**

<sup>1</sup>School of Electrical, Computer and Energy Engineering, Arizona State University, Tempe, AZ 85287, USA

<sup>2</sup>Facebook Inc., Menlo Park, CA 94025, USA

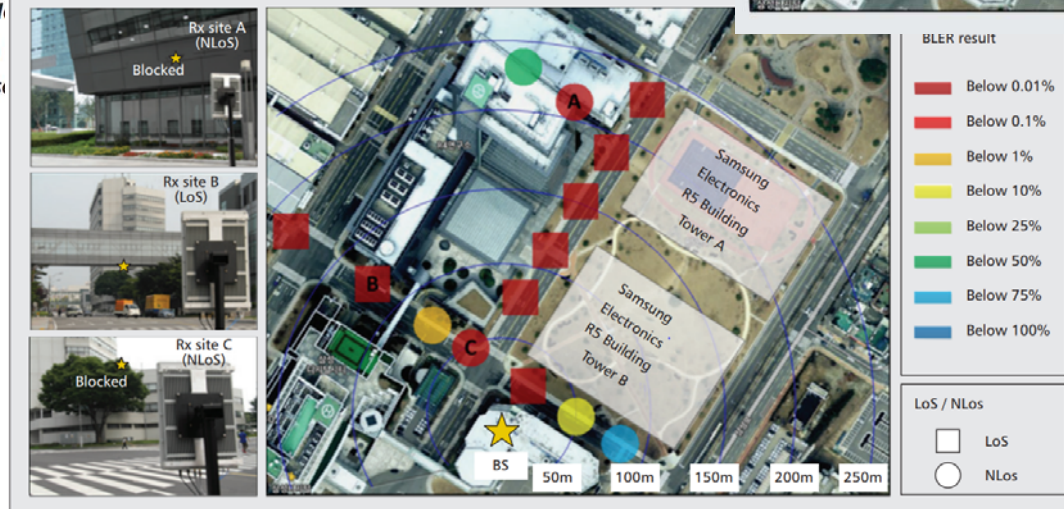
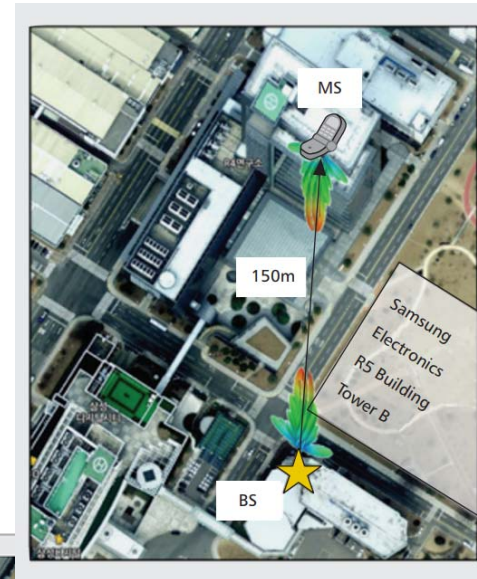
Corresponding author: Ahmed Alkhateeb (alkhateeb@asu.edu)

# Potential research directions

- Hardware implementation and testing

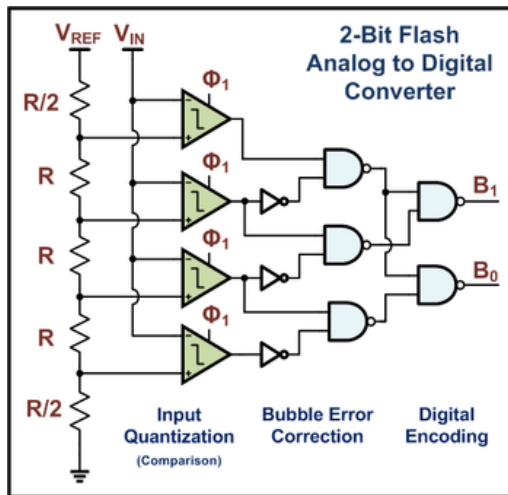
## Millimeter-Wave Beamforming as an Enabling Technology for 5G Cellular Communications: Theoretical Feasibility and Prototype Results

Wonil Roh, Ji-Yun Seol, JeongHyeon Cheun, Samsung Research Asia  
 Farshid Aryanfar, Samsung Research Asia



# Potential research directions

## ➤ Hybrid precoding with low-precision ADCs



## Hybrid Architectures With Few-Bit ADC Receivers: Achievable Rates and Energy-Rate Tradeoffs

Jianhua Mo, *Member, IEEE*, Ahmed Alkhateeb, *Member, IEEE*, Shadi Abu-Surra, *Member, IEEE*,  
and Robert W. Heath, Jr., *Fellow, IEEE*

Performance evaluation with  
tractable quantization models

High-precision ADCs at mm-wave  
frequencies are extremely expensive

## ❖ Our own results

- X. Yu, J.-C. Shen, **J. Zhang**, and K. B. Letaief, “Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems,” *IEEE J. Sel. Topics Signal Process., Special Issue on Signal Process. for Millimeter Wave Wireless Commun.*, vol. 10, no. 3, pp. 485-500, Apr. 2016. (**The 2018 SPS Young Author Best Paper Award**)
- X. Yu, **J. Zhang**, and K. B. Letaief, “Alternating minimization for hybrid precoding in multiuser OFDM mmWave Systems,” in *Proc. Asilomar Conf. on Signals, Systems, and Computers*, Pacific Grove, CA, Nov. 2016. (**Invited Paper**)
- X. Yu, **J. Zhang**, and K. B. Letaief, “Partially-connected hybrid precoding in mm-wave systems with dynamic phase shifter networks,” in *Proc. IEEE Int. Workshop Signal Process. Adv. Wireless Commun. (SPAWC)*, Sapporo, Japan, Jul. 2017.
- X. Yu, **J. Zhang**, and K. B. Letaief, “Hybrid precoding in millimeter wave systems: How many phase shifters are needed?” in *Proc. IEEE Global Commun. Conf. (Globecom)*, Singapore, Dec. 2017. (**Best Paper Award**)
- X. Yu, **J. Zhang**, and K. B. Letaief, “A hardware-efficient analog network structure for hybrid precoding in millimeter wave systems,” *IEEE J. Sel. Topics Signal Process., Special Issue on Hybrid Analog-Digital Signal Processing for Hardware-Efficient Large Scale Antenna Arrays*, vol. 12, no. 2, pp. 282-297, May 2018.

## ❖ References in this tutorial

- X. Zhang, A. F. Molisch, and S.-Y. Kung, “Variable-phase-shift-based RF-baseband codesign for MIMO antenna selection,” *IEEE Trans. Signal Process.*, vol. 53, no. 11, pp. 4091-4103, Nov. 2005.
- E. Zhang and C. Huang, “On achieving optimal rate of digital precoder by RF-baseband codesign for MIMO systems,” in *Proc. IEEE 80th Veh. Technol. Conf. (VTC2014-Fall)*, Vancouver, BC, Sep. 2014, pp. 1-5.
- O. E. Ayach, S. Rajagopal, S. Abu-Surra, Z. Pi, and R. W. Heath, Jr., “Spatially sparse precoding in millimeter wave MIMO systems,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 3, pp. 1499-1513, Mar. 2014.
- N. Boumal, B. Mishra, P. A. Absil, R. Sepulchre, “Manopt, a Matlab toolbox for optimization on manifolds,” *The Journal of Machine Learning Research* vol. 15, no. 1, pp. 1455-1459, 2014.
- X. Gao, L. Dai, S. Han, C.-L. I, and R. W. Heath, Jr., “Energy-efficient hybrid analog and digital precoding for mmWave MIMO systems with large antenna arrays,” *IEEE J. Sel. Areas in Commun.*, vol. 34, no. 4, pp. 998-1009, Apr. 2016.



## ❖ References in this tutorial

- F. Sofrabi and W. Yu, “Hybrid digital and analog beamforming design for large-scale antenna arrays,” *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 501-513, Apr. 2016.
- Y. Lee, C.-H. Wang, and Y.-H. Huang, “A hybrid RF/baseband precoding processor based on parallel-index-selection matrix-inversion-bypass simultaneous orthogonal matching pursuit for millimeter wave MIMO systems,” *IEEE Trans. Signal Process.*, vol. 63, no. 2, pp. 305–317, Jan. 2015.
- L. Liang, W. Xu, and X. Dong, “Low-complexity hybrid precoding in massive multiuser MIMO systems,” *IEEE Wireless Commun. Lett.*, vol. 3, no. 6, pp. 653–656, Dec. 2014.
- F. Sofrabi and W. Yu, “Hybrid Analog and Digital Beamforming for mmWave OFDM Large-Scale Antenna Arrays,” *IEEE J. Sel. Areas Commun.*, vol. 35, no. 7, pp. 1432-1443, July 2017.
- Z. Feng, S. Fu, T. Ming, and D. Liu, “Multichannel continuously tunable microwave phase shifter with capability of frequency doubling,” *IEEE Photon. J.*, vol. 6, no. 1, pp. 1–8, Feb. 2014.

## ❖ References in this tutorial

- A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., “Channel estimation and hybrid precoding for millimeter wave cellular systems,” *IEEE J. Sel. Topics Signal Process.*, vol. 8, no. 5, pp. 831-846, Oct. 2014.
- Z. Gao, C. Hu, L. Dai, and Z. Wang, “Channel estimation for millimeter-wave massive MIMO with hybrid precoding over frequency-selective fading channels,” *IEEE Commun. Lett.*, vol. 20, no. 6, pp. 1259-1262, June 2016.
- A. Alkhateeb, O. El Ayach, G. Leus, and R. W. Heath, Jr., “Hybrid precoding for millimeter wave cellular systems with partial channel knowledge,” in *Proc. Inf. Theory Applications Workshop (ITA)*, San Diego, CA, 2013, pp. 1-5.
- M. N. Kulkarni, A. Ghosh, and J. G. Andrews, “A comparison of MIMO techniques in downlink millimeter wave cellular networks with hybrid beamforming,” *IEEE Trans. Commun.*, vol. 64, no. 5, pp. 1952-1967, May 2016.
- X. Gao, L. Dai, Y. Sun, S. Han, and C.-L. I, “Machine learning inspired energy-efficient hybrid precoding for mmWave massive MIMO systems,” in *Proc. IEEE Int. Conf. Commun. (ICC)*, Paris, France, May, 2017, pp. 1-6.

## ❖ References in this tutorial

- A. Alkhateeb, S. Alex, P. Varkey, Y. Li, Q. Qu, and D. Tujkovic, “Deep learning coordinated beamforming for highly-mobile millimeter wave systems,” *IEEE Access*, vol. 6, pp. 37328-37348, 2018.
- W. Roh et al., “Millimeter-wave beamforming as an enabling technology for 5G cellular communications: theoretical feasibility and prototype results,” *IEEE Commun. Mag.*, vol. 52, no. 2, pp. 106-113, Feb. 2014.
- J. Mo, A. Alkhateeb, S. Abu-Surra and R. W. Heath, “Hybrid architectures with few-bit ADC receivers: Achievable rates and energy-rate tradeoffs,” *IEEE Trans. Wireless Commun.*, vol. 16, no. 4, pp. 2274-2287, Apr. 2017.
- Y. Zeng and R. Zhang, “Millimeter Wave MIMO With Lens Antenna Array: A New Path Division Multiplexing Paradigm,” *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1557-1571, Apr. 2016.
- X. Gao, L. Dai, S. Han, C.-L. I, and X. Wang, “Reliable beamspace channel estimation for millimeter-wave massive MIMO systems with lens antenna array,” *IEEE Trans. Wireless Commun.*, vol. 16, no. 9, pp. 6010-6021, Sep. 2017.



## ❖ Other references

- J. Brady, N. Behdad, and A. M. Sayeed, “Beamspace MIMO for millimeter-wave communications: System architecture, modeling, analysis, and measurements,” *IEEE Trans. Antennas Propag.*, vol. 61, no. 7, pp. 3814–3827, Jul. 2013.
- J. Singh and S. Ramakrishna, “On the feasibility of codebook-based beamforming in millimeter wave systems with multiple antenna arrays,” *IEEE Trans. Wireless Commun.*, vol. 14, no. 5, pp. 2670–2683, May 2015.
- S. Park, A. Alkhateeb, and R.W. Heath, Jr., “Dynamic subarrays for hybrid precoding in wideband mmWave MIMO systems,” *IEEE Trans. Wireless Commun.*, vol. 16, no. 5, pp. 2907–2920, May 2017.
- T. E. Bogale, L. B. Le, A.Haghighat, and L.Vandendorpe, “On the number of RF chains and phase shifters, and scheduling design with hybrid analog digital beamforming,” *IEEE Trans. Wireless Commun.*, vol. 15, no. 5, pp. 3311–3326, May 2016.

## ❖ Other references

- A. Alkhateeb and R.W. Heath, Jr., “Frequency selective hybrid precoding for limited feedback millimeter wave systems,” *IEEE Trans. Commun.*, vol. 64, no. 5, pp. 1801–1818, May 2016.
- R. Mendez-Rial, C. Rusu, N. Gonzalez-Prelcic, A. Alkhateeb, and R. W. Heath, Jr., “Hybrid MIMO architectures for millimeter wave communications: Phase shifters or switches?” *IEEE Access*, vol. 4, pp. 247–267, Jan. 2016.
- W. Ni and X. Dong, “Hybrid block diagonalization for massive multiuser MIMO systems,” *IEEE Trans. Commun.*, vol. 64, no. 1, pp. 201–211, Jan. 2016.
- T. E. Bogale and L. B. Le, “Beamforming for multiuser massive MIMO systems: Digital versus hybrid analog-digital,” in *Proc. IEEE Global Commun. Conf.*, Austin, TX, USA, Dec. 2014, pp. 4066–4071.

*Thanks*

For more information and **Matlab codes**:

<http://www.ece.ust.hk/~eejzhang/>