Performance Analysis of SDMA in Multicell Wireless Networks

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***Abstract*—Multi-antenna transmission, or MIMO, is a major enabling technique for broadband cellular networks. The current implementation, however, is mainly for the point-to-point link, and its potential for Space-Division Multiple Access (SDMA) has not been fully exploited. In this paper, we will analytically evaluate the performance of SDMA in multicell networks based on a spatial random network model, where both the base stations (BSs) and users are modeled as two independent Poisson point processes. The main difficulty is the evaluation of the interference distribution, for which we propose a novel BS grouping approach that leads to a closed-form expression for the network area spectral efficiency. We find that the number of active users (U) served with SDMA is critical, as it affects the spatial multiplexing gain, the aggregated interference, and the diversity gain for each user. The optimal value of U can be selected based on our analytical result, with which SDMA is shown to outperform both the single-user beamforming and full-SDMA for which U is the same as the number of BS antennas. In particular, it is shown that the performance gain of SDMA is higher when the BS density is relatively small compared to the user density, but the optimal value of U is almost the same for different scenarios, which is close to half of the BS antenna number.**

***Keywords*: Cellular networks, Poisson point process, SDMA.**

I. Introduction

Multi-antenna (or MIMO) techniques have been widely adopted in wireless networks, such as WiMAX and LTE [1], which help to improve both the transmission reliability and the spectral efficiency. While current implementations mainly support single-user MIMO, multi-user MIMO, also called as Space-Division Multiple Access (SDMA), has also been investigated to further improve system performance [2]. Compared with single-user MIMO techniques, SDMA allows each base station (BS) to serve multiple mobile users simultaneously, which will significantly improve the network spectral efficiency. However, there are practical obstacles for realizing the performance gains of SDMA, such as channel state information (CSI) acquisition, and the enhanced interference, including intra-cell inter-user interference and inter-cell interference. The impact of imperfect CSI in SDMA systems has received lots of attention [3], [4]. In particular, it has been shown that the residual inter-user interference due to imperfect CSI will significantly degrade the performance of SDMA, and the number of served users should be carefully selected based on the CSI accuracy [5]. However, previous works mainly considered the single-cell scenario, while inter-cell interference was ignored. In this paper, we will evaluate the performance of SDMA in multicell networks.

Due to the complexity of the network topology, and the effect of multi-path propagation, it is difficult to analytically evaluate the performance of multicell networks. Recently, Andrews *et al.* proposed a tractable cellular network model where BSs are distributed as a spatial Poisson point process (PPP) [6], and it was demonstrated that this framework can accurately model the multicell network and is amenable for performance analysis. Actually, such random network model has been widely used in wireless ad hoc networks. In particular, SDMA was investigated with this model in [7], with a focus on the effect of limited feedback. However, the interference model in [7] was different from that in cellular networks. While the interfering nodes can be arbitrarily close in wireless ad hoc networks, the interfering BSs in the cellular network will be farther than the home BS [6]. Moreover, it was assumed in [7] that each transmitter can choose to serve an arbitrary number of users with SDMA, which is not valid in cellular networks, as the number of users in each cell depends on the user distribution. So far, it is not clear how the number of users served by SDMA in the cellular network will affect the system performance. On one hand, serving more users simultaneously will increase the number of transmitted data streams. On the other hand, more active users result in higher interference and less diversity gain for each user.

In this paper, we will evaluate the performance of SDMA in the downlink cellular network with a random spatial network model, where BSs and users are modeled as two independent homogeneous PPPs. Therefore, the number of users in each cell is different. We consider the following U-SDMA system: if there are N users in a cell, its BS will serve $\min(N,U)$ users, so the number of active users in each cell depends on the network realization. For a given value of U, we will analytically evaluate the area spectral efficiency (ASE) of the network, and then select the optimal value that maximizes the ASE, denoted as $U^\star$. Based on the numerical evaluation, we have the following observations:

1) When the BS density is much smaller than the user density, the ASE with $U^\star$-SDMA increases linearly with the number of BS antennas M. It also outperforms both single-user beamforming ($U=1$) and full-SDMA ($U=M$).
2) When the BS density and the user density are com-

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parable, the performance of full-SDMA is close to that of $U^\star$-SDMA, while both outperform single-user beamforming. But the performance gain diminishes and converges to a constant for large values of M.
3) The optimal U (i.e., $U^\star$) is always close to $\frac{M}{2}$, and it is almost the same for different BS densities.

In different scenarios, $U^\star$-SDMA can easily more than double the ASE compared to the single-user beamforming. Thus, our results demonstrate the advantage of SDMA in multicell networks and indicate the importance of selecting U.

The outline of this paper is as follows. In Section II, we present the system model and the performance metric. The derivation of the ASE is presented in Section III. Then in Section IV, we evaluate the ASE numerically, while Section V concludes the paper.

II. System model

In this section, we will first describe the random spatial model for cellular networks, and then present the ASE as the system performance metric.

A. Network Model

We consider a cellular network, where BSs and mobile users are distributed according to two independent PPPs in $\mathbb{R}^2$, denoted as Ψ_b and Ψ_u, respectively. Denote the BS density as λ_b and the user density as λ_u. We assume that each user is served by the nearest BS, which comprises a Voronoi tessellation relative to λ_b. Hence, the shape of each cell is irregular, as shown in Fig. 1. Due to the independent locations of BSs and users, the number of users in each cell is in general different. Denote N as the number of users in a typical cell, then N could be any non-negative integer.

We assume each BS is equipped with M transmit antennas, while each user has a single receive antenna. As different cells have different numbers of users, we consider the following U-SDMA transmission scheme: With N users in the cell, the BS will serve $\min(N, U)$ users simultaneously, where $1 \le U \le M$. Subsequently, the users being served in this cell are called *active users*, whose number is denoted as $u \triangleq \min(N, U)$. We have the following three different situations due to different values of N, which are also demonstrated in Fig. 1:

- For $N = 0$, i.e., no user in this cell, the BS will not transmit any signal and it is called an *inactive* BS.
- For $N \le U$, all of these N users will be simultaneously served by the BS.
- For $N > U$, the BS will randomly choose U users to serve.

BS cooperation is not considered in this paper as we assume that the capacity of the backhaul links between different BSs is limited. Universal frequency reuse is assumed, and thus each user not only receives information from its home BS, but also suffers from interference from all the other active BSs. Zero-forcing precoding is applied at each BS, due to its simplicity and tractability [3]–[5]. We assume that perfect CSI of the channels to its own users is available at each BS, so there is no intra-cell interference. In this paper we will focus on the effect of inter-cell interference while the effect of imperfect CSI will be investigated in the future work.

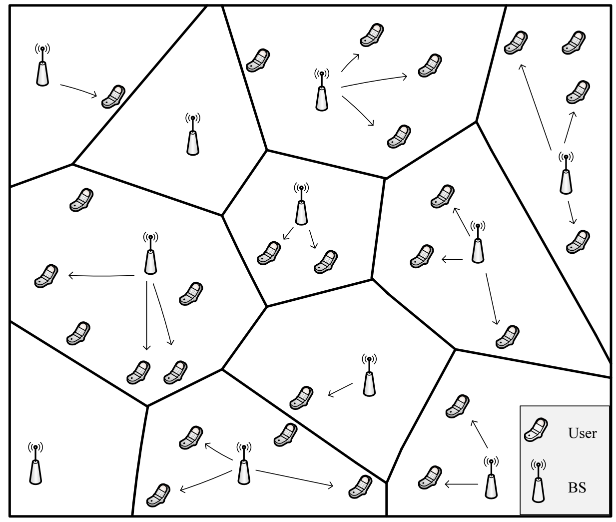

Fig. 1. A sample network where BSs and users are distributed as two independent PPPs. Each user is connected to the closest BS. We set $U = 3$ in this example, so the number of active users u can be 1, 2, and 3.

For the channel model, we consider Rayleigh fading with pathloss. If the BS needs to serve multiple users, equal power allocation is adopted. Therefore, for a typical active user, and assuming there are u_0 active users in its cell, the received signal-to-interference plus noise ratio (SINR) is given by

$$\mathrm{SINR}(u_0) = \frac{\frac{P_{\mathrm{t}}}{u_0} \bar{g}_{00} r_0^{-\alpha}}{\sum_{i \in \bar{\Psi}_b} \frac{P_{\mathrm{t}}}{u_i} \bar{g}_i R_{i0}^{-\alpha} + \sigma_n^2}, \tag{1}$$

where P_{t} is the BS transmit power, $\bar{g}_{00}$ is the channel gain from the signal transmission link, and r_0 is the distance between the user to its home BS. The set of interfering BSs is denoted as $\bar{\Psi}_b$, and u_i is the number of active users in the ith cell, $\bar{g}_i$ is the ith interfering channel gain, and R_{i0} is the distance from the ith interfering BS to the user. The noise variance is denoted as σ_n^2. With zero-forcing precoding, it is shown in [8] that both $\bar{g}_{00}$ and $\bar{g}_i$ are Gamma distributed, i.e., $\bar{g}_{00} \sim \mathrm{Gamma}(M - u_0 + 1, 1)$ and $\bar{g}_i \sim \mathrm{Gamma}(u_i, 1)$.

Remark 1: Note that u_0 and u_i are randomly distributed between 1 and U. Therefore, setting different values of U will affect both the signal power and the interference distribution. However, the net effect is not obvious and will be evaluated later.

B. The Network Performance Metric

In this paper, we apply ASE as the system performance metric. We consider fixed-rate transmission, where the ASE is mainly determined by the successful transmission probability, which is defined as the probability that the received SINR is greater than a given threshold $\hat{\gamma}$, i.e.,

$$p_{\mathrm{s}} \triangleq \Pr(\mathrm{SINR} \ge \hat{\gamma}), \tag{2}$$

where SINR is given in (1).

TABLE I
KEY NOTATIONS AND SYMBOLS USED IN THE PAPER

Symbol	Definition/Explanation
λ_b	BS density
λ_u	User density
ρ	BS-user density ratio, i.e., $\frac{\lambda_b}{\lambda_u}$
α	Pathloss exponent
$\hat{\gamma}$	SINR threshold
M	Number of transmit antennas per BS
N	Number of users in a cell
U	Maximum number of active users ($U \leq M$)
u	Number of active users ($1 \leq u \leq U$)
$p_{\mathrm{SU}}(u)$	Probability that there are u active users
$p_{\mathrm{s}}(u)$	Successful transmission probability of a user in a cell with u active users
R_{a}	Area spectral efficiency

The ASE is defined as the average number of successfully transmitted bits per sec·Hz·unit-area [9]. Given a cell with u active users, the BS throughput is $up_{\mathrm{s}}(u)\log_2(1+\hat{\gamma})$. Therefore, the ASE of the network is

$$R_{\mathrm{a}} \triangleq \lambda_b \mathrm{E}_u \left[up_{\mathrm{s}}(u) \log_2(1+\hat{\gamma}) \right]. \tag{3}$$

To determine the network throughput, we need to obtain the distribution of the number of active users in a cell, denoted as $p_{SU}(u)$, and the successful transmission probability for a given u, which will be derived in the following section. For reference, Table I lists the key notations and symbols used in this paper.

III. Area Spectral Efficiency Analysis

In this section, we will first derive $p_{\mathrm{SU}}(u)$, then derive the successful transmission probability $p_{\mathrm{s}}(u)$. By combining these results, we will lastly present the closed-form expression of the ASE.

A. The Distribution of the Number of Active Users

We first obtain the distribution of the number of users in a cell. As users are distributed spatially as a PPP, the number of users in a cell with cell size X follows a Poisson distribution. The distribution of the cell size X was obtained by the Monte Carlo method in [10] as

$$f_X(x) = \frac{\mu^{\mu}}{\Gamma(\mu)} x^{\mu-1} e^{-\mu x}, \tag{4}$$

where $\mu = 3.5$ is a constant obtained through data fitting, X denotes the cell size normalized by $\frac{1}{\lambda_b}$, and $\Gamma(\cdot)$ is the Gamma function. The distribution of the number of users in a cell is then provided in the following lemma.

Lemma 1: The probability mass function (PMF) of the number of users in a cell is given by

$$p_N(n) = \frac{\mu^{\mu}\Gamma(n+\mu)\rho^{-n}}{\Gamma(\mu)\, n! \left(\frac{1}{\rho}+\mu\right)^{n+\mu}}, \tag{5}$$

for $n \in \mathbb{N}$, and $\rho \triangleq \frac{\lambda_b}{\lambda_u}$ is the BS-user density ratio.

Proof: See Appendix A. ∎

Based on this distribution, the probability of the number of active users in a cell is given by

$$p_{\mathrm{SU}}(u) = \begin{cases} p_N(u) & 0 \leq u \leq U-1 \\ \sum_{i=U}^{\infty} p_N(i) & u = U. \end{cases} \tag{6}$$

Remark 2: Note that in [7] or other related works, the number of users served by a transmitter can be an arbitrary number and is the same for all the transmitters, which means that u is a deterministic number, and thus the ASE only depends on the successful transmission probability. However, in our system model, due to the BS and user distributions, the value of u in each cell is random, with the distribution given in (6). Subsequently, the ASE in (3) can be written as

$$R_{\mathrm{a}} = \lambda_b \sum_{u=1}^{U} up_{\mathrm{s}}(u) p_{\mathrm{SU}}(u) \log_2(1+\hat{\gamma}). \tag{7}$$

B. The Successful Transmission Probability

In this subsection, a closed-form expression of the successful transmission probability will be derived. We will consider an interference-limited scenario and ignore additive noise in the following analysis. Later we will justify such an approximation through simulation. The received SIR of a typical user in the cell with u_0 active users is given by

$$\mathrm{SIR}(u_0) = \frac{\frac{1}{u_0}\bar{g}_{00} r_0^{-\alpha}}{\sum_{i\in\bar{\Psi}_b} \frac{1}{u_i}\bar{g}_i R_{i0}^{-\alpha}}. \tag{8}$$

The major difficulty in the derivation is that the interfering channel gains $\bar{g}_i$ are not identically distributed, because each interfering BS may serve a different number of users. Specifically, $\bar{g}_i \sim \mathrm{Gamma}(u,1)$ where u could be from 1 to U, and the distribution of u is shown in (6). To the best of our knowledge, there is no previous result for the successful transmission probability with SIR given as (8). In this paper, we propose a novel method to resolve this issue. We shall divide all the interfering BSs into U subsets, denoted as Φ_u for $u = 1, 2, \ldots, U$, so that each BS in the subset Φ_u is serving u users. In this way, we divide all the interfering BSs into different groups, and the interfering channel gains from the BSs in the group Φ_u follow independent and identically distributed (i.i.d.) Gamma distribution with the same shape parameter u. However, the network is still complex and intractable, as the numbers of users in different cells are coupled due to the BS/user distribution, so the interference distribution is still difficult to obtain. In the following, we further assume that the number of active users in one cell is independent of each other so that each BS group Φ_u follows a homogeneous PPP distribution with density $\lambda_b P_{\mathrm{SU}}(u)$. This approximation was adopted in [11], [12] for the single-user transmission, where it was shown to be quite accurate. We will evaluate this approximation in Section IV through simulation. Based on the BS grouping, the received SIR in (8) can be rewritten as

$$\mathrm{SIR}(u_0) = \frac{g_{00} r_0^{-\alpha}}{\sum_{u=1}^{U} \sum_{j\in\Phi_u} g_{uj} R_{uj}^{-\alpha}}, \tag{9}$$

where R_{uj} is the distance from the jth BS in the subset Φ_u, $g_{00} \sim \text{Gamma}\left(M-u_0+1, \frac{1}{u_0}\right)$ and $g_{uj} \sim \text{Gamma}\left(u, \frac{1}{u}\right)$.

Based on the SIR expression (9), the successful transmission probability is given by

$$p_{\text{s}}\left(u_0\right) = \Pr\left(\frac{g_{00} r_0^{-\alpha}}{\sum_{u=1}^{U} \sum_{j\in\Phi_u} g_{uj} R_{uj}^{-\alpha}} \geq \hat{\gamma}\right). \quad (10)$$

Then using the cumulative distribution function of the Gamma distribution g_{00}, Eq. (10) can be calculated as

$$p_{\text{s}}\left(u_0\right) = \mathrm{E}_{r_0}\left[\sum_{l=0}^{M-u_0} \mathrm{E}_I\left[\frac{1}{l!}\left(sI\right)^l e^{-sI}\right]\right], \quad (11)$$

where $s = u_0 r_0^{\alpha}$ and $I = \hat{\gamma}\sum_{u=1}^{U}\sum_{j\in\Phi_u} g_{uj} R_{uj}^{-\alpha}$. Denote the Laplace transform of I as $\mathcal{L}_I\left(s\right)$, i.e., $\mathcal{L}_I\left(s\right) = \mathrm{E}_I\left[e^{-sI}\right]$. Then, following the property of the Laplace transform $\mathrm{E}_I\left[I^l e^{-sI}\right] = \left(-1\right)^l \mathcal{L}_I^{(l)}\left(s\right)$, where $\mathcal{L}_I^{(l)}\left(s\right)$ is the lth derivative of $\mathcal{L}_I\left(s\right)$, the successful transmission probability can be written as

$$p_{\text{s}}\left(u_0\right) = \mathrm{E}\left[\sum_{l=0}^{M-u_0} \frac{1}{l!} s^l \left(-1\right)^l \mathcal{L}_I^{(l)}\left(s\right)\right]. \quad (12)$$

In [12], the lth derivative of $\mathcal{L}_I\left(s\right)$ was derived, where all the interfering channel gains g_{uj} follow i.i.d. exponential distribution, so the result cannot be directly extended to our case. Following a similar approach as in [12], we are able to derive the following closed-form expression of the successful transmission probability, given in the following proposition.

Proposition 1: The successful transmission probability for a typical user given there are u_0 active users in the home cell is given by

$$p_{\text{s}}\left(u_0\right) = \left\|\mathbf{Q}\left(u_0\right)^{-1}\right\|_1, \quad (13)$$

where $\left\|\cdot\right\|_1$ is the L_1 induced matrix norm (i.e., $\left\|\mathbf{A}\right\|_1 = \max_{1\leq j\leq n}\sum_{i=1}^{m}\left|a_{ij}\right|$ for $\mathbf{A} \in \mathbb{R}^{m\times n}$), and $\mathbf{Q}\left(u_0\right)$ is an $(M-u_0+1)\times(M-u_0+1)$ lower triangular Toeplitz matrix, given by

$$\mathbf{Q}\left(u_0\right) = \begin{bmatrix} 1+k_0 & 0 & & & \\ -k_1 & 1+k_0 & 0 & & \\ -k_2 & -k_1 & 1+k_0 & \ddots & \\ \vdots & \vdots & \ddots & \ddots & 0 \\ -k_{M-u_0} & \cdots & -k_2 & -k_1 & 1+k_0 \end{bmatrix},$$

in which k_0 and k_j for $j \geq 1$ depend on u_0, given by

$$k_0\left(u_0\right) = \sum_{u=1}^{U} p_{\text{SU}}\left(u\right)\hat{\gamma}^{\frac{2}{\alpha}} \int_{\hat{\gamma}^{-\frac{2}{\alpha}}}^{\infty} 1 - \frac{1}{\left(1+\frac{u_0}{u}v^{-\frac{\alpha}{2}}\right)^u} dv, \quad (14)$$

and

$$k_j\left(u_0\right) = \sum_{u=1}^{U} p_{\text{SU}}\left(u\right)\frac{\left(u\right)_j}{j!}\hat{\gamma}^{\frac{2}{\alpha}} \int_{\hat{\gamma}^{-\frac{2}{\alpha}}}^{\infty} \frac{\left(\frac{u_0}{u}v^{-\frac{\alpha}{2}}\right)^j dv}{\left(1+\frac{u_0}{u}v^{-\frac{\alpha}{2}}\right)^{u+j}}, \quad (15)$$

for $j \geq 1$, where $\left(u\right)_j$ is the Pochhammer symbol (i.e., $\left(u\right)_j = \frac{\Gamma(u+j)}{\Gamma(u)}$).

Proof: The proof is omitted due to space limitation. ■

There are certain structures in expression (13) which can assist our analysis. For example, it was shown in [13] that the inverse matrix of the lower triangular Toeplitz matrix is still a lower triangular Toeplitz matrix, so the properties of the Toeplitz matrix can be used to analyze the matrix $\mathbf{Q}\left(u_0\right)^{-1}$. Some analytical results in [14] may also be applied here. More analytical investigation will be pursued in future work.

C. The Area Spectral Efficiency

By substituting the successful transmission probability (13) into Eq. (7), the network ASE is given by

$$R_{\text{a}} = \lambda_b \sum_{u_0=1}^{U} u_0 \left\|\mathbf{Q}\left(u_0\right)^{-1}\right\|_1 p_{\text{SU}}\left(u_0\right)\log_2\left(1+\hat{\gamma}\right). \quad (16)$$

Note that the ASE expression in (16) depends on the BS density λ_b, the user density λ_u, the number of BS antennas M, and the value of U. Through the derivation, we have already seen that increasing U has various effects on the performance. The number of data streams, i.e., the spatial multiplexing gain, will be increased, while the interference distribution will be changed and the diversity gain for each user will be decreased. With (16), we can numerically evaluate the net effect of varying U, and the optimal value $U^\star$ that maximizes ASE can be determined by search, which is affordable as the number of BS antennas in practical systems is relatively small, e.g., $M = 4$ for LTE while M is up to 8 for LTE-Advanced.

IV. Numerical Evaluation

In this section, we will numerically evaluate the result in Eq. (16), and key observations will be made. For the simulation results, the additive noise is included to test the interference-limited approximation. Fig. 2 shows the network ASE with different U. Note that for $U = 1$, each BS will randomly choose one user to serve at each timeslot, which can be regarded as the single-user beamforming (SU-BF) investigated in [12]. On the other hand, for $U = M$, the BS will use M antennas to serve a maximum of M users at each timeslot, denoted as full-SDMA.[1] From Fig. 2, we see that our analytical results fit the simulation curves very well, which shows the accuracy of our interference approximation and also validates the interference-limited assumption. Moreover, we can see that by selecting a proper U, the network ASE can always be improved compared with the SU-BF or full-SDMA. In particular, by selecting the optimal value $U^\star$, the $U^\star$-SDMA can easily more than double the ASE compared to the SU-BF. We also see that the BS-user density ratio plays a significant role. When $\lambda_b \ll \lambda_u$, i.e., the lower curve, the optimal $U^\star$ is around $\frac{M}{2}$, and its performance is much better than either $U = 1$ or $U = M$. While when the BS density and the user density are comparable, i.e., $\lambda_b \sim \lambda_u$ as for the upper curve, the variation of U has little effect on the ASE for medium to

[1]This notation is slightly different from others, where full-SDMA means serving M users with M antennas, as in [7]. In our case, the number of active users is $\min\left(N, M\right)$, which may be less than M.

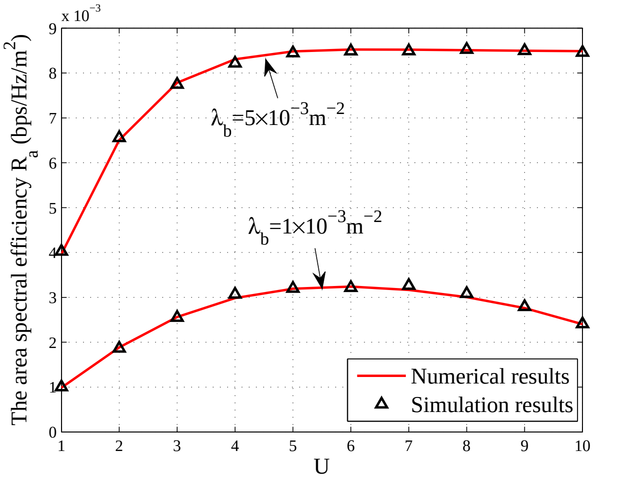

Fig. 2. The ASE with different values of U, with $M=10$, $\alpha=4$, $\hat{\gamma}=1$, $\lambda_u=10^{-2}\mathrm{m}^{-2}$, $P_{\mathrm{t}}=6.31\mathrm{W}$ [15], and the noise power is considered in simulation with $\sigma_n^2=-97.5\mathrm{dBm}$ [16].

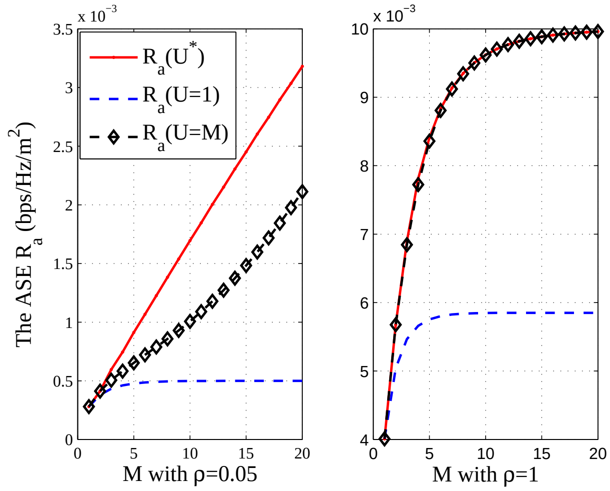

Fig. 3. The ASE with different values of M, with $\lambda_u=10^{-2}\mathrm{m}^{-2}$, $\alpha=4$, $\hat{\gamma}=1$.

large values of U. This is because in this case the number of active users is limited by the number of users in the cell, but not U. Note that the effect of the BS/user density cannot be revealed from previous works as [7].

Next, we compare the ASE with different values of M in Fig. 3, from which we can make the following observations:

- When $\lambda_b \ll \lambda_u$, for $U^\star$-SDMA, the network ASE increases linearly with respect to the number of BS antennas M, and it provides significant throughput gain over both the SU-BF and full-SDMA, as shown in the left figure.
- When $\lambda_b \sim \lambda_u$, $U^\star$-SDMA and full-SDMA provide similar performance, both of which outperform SU-BF. Moreover, the performance gain diminishes as M increases, and the ASE tends to converge to a constant, as shown in the right figure.

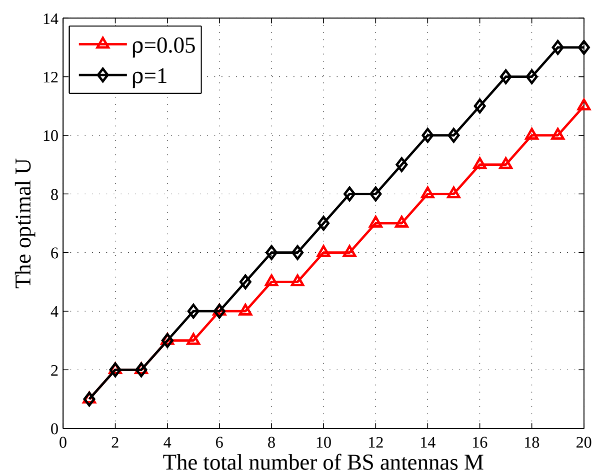

Fig. 4. The optimal U with different values of M, with $\lambda_u=10^{-2}\mathrm{m}^{-2}$, $\alpha=4$, $\hat{\gamma}=1$.

In Fig. 4, we plot the optimal $U^\star$ for different numbers of BS antennas. We see that $U^\star$ is always very close to $\frac{M}{2}$, and it is almost the same for different BS densities. This phenomenon always exists as we try different parameter sets, and further investigation is needed to find out the reason. Note that both Fig. 3 and Fig. 4 are based on our analytical result (16), and the computational complexity would be extremely high if we instead try to get the same results by simulation.

V. Conclusions

In this paper, we analytically evaluated the performance of SDMA in multicell networks, based on the spatial random network model. The main difficulty lied in the complicated interference distribution, which is due to the random number of users in each cell. We proposed a novel method to tackle the problem by dividing all the interfering BSs into different groups, where BSs in the same group enjoy the same distribution of the interfering channel. Based on this method, we derived a closed-form expression of the network ASE, from which the optimal U, denoted as $U^\star$, that maximizes the ASE can be determined. Numerical results demonstrated the advantages of $U^\star$-SDMA over both the SU-BF and full-SDMA. Moreover, it was shown that $U^\star$ is close to the half of the BS antenna number and is insensitive to the BS density.

Appendix

A. *Proof of Lemma 1*

The PMF of the number of users in a cell is given by

$$p_N(n)=\int_0^\infty \Pr\left(N=n \mid X=x\right) f_X(x)\,dx. \tag{17}$$

Since the user distribution is a PPP, the above equation can be rewritten as

$$p_N(n) = \int_0^\infty \frac{\left(\lambda_u \frac{x}{\lambda_b}\right)^n}{n!} \exp\left(-\lambda_u \frac{x}{\lambda_b}\right) f_X(x)\, dx. \quad (18)$$

By substituting (4) into (18), we obtain Eq. (5).

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