Hybrid Precoding for Millimeter Wave MIMO Systems
- Algorithm Design and Hardware Implementation

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Outline

- **Background and Motivation**
- **Introduction to Hybrid Precoding**
- **Algorithm Design & Hardware Implementation**
  - Single Phase Shifter (SPS) Implementation
  - Double Phase Shifter (DPS) Implementation
  - Fixed Phase Shifter (FPS) Implementation
- **Conclusions**

Hybrid Precoding for mm-Wave MIMO Systems

Background and Motivation

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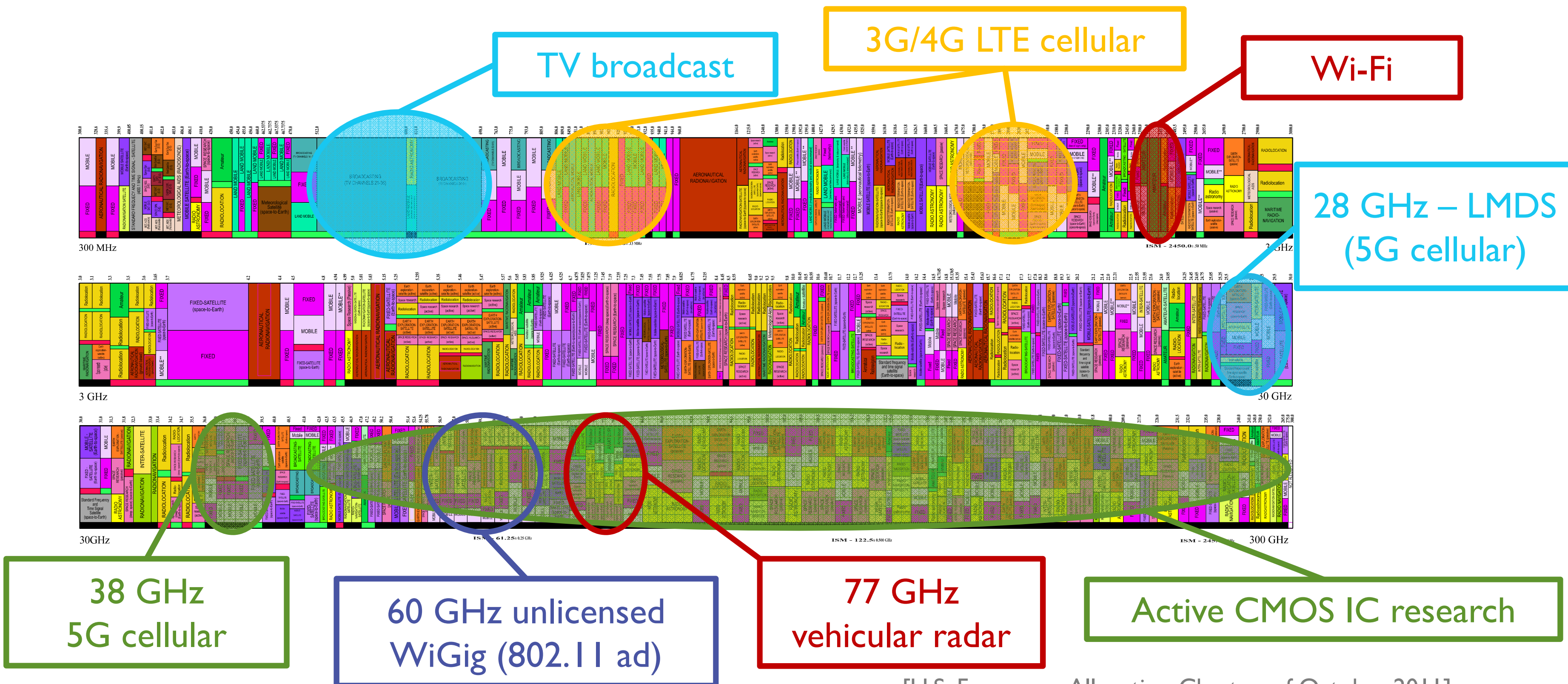
- Spectrum crunch: A fundamental bottleneck

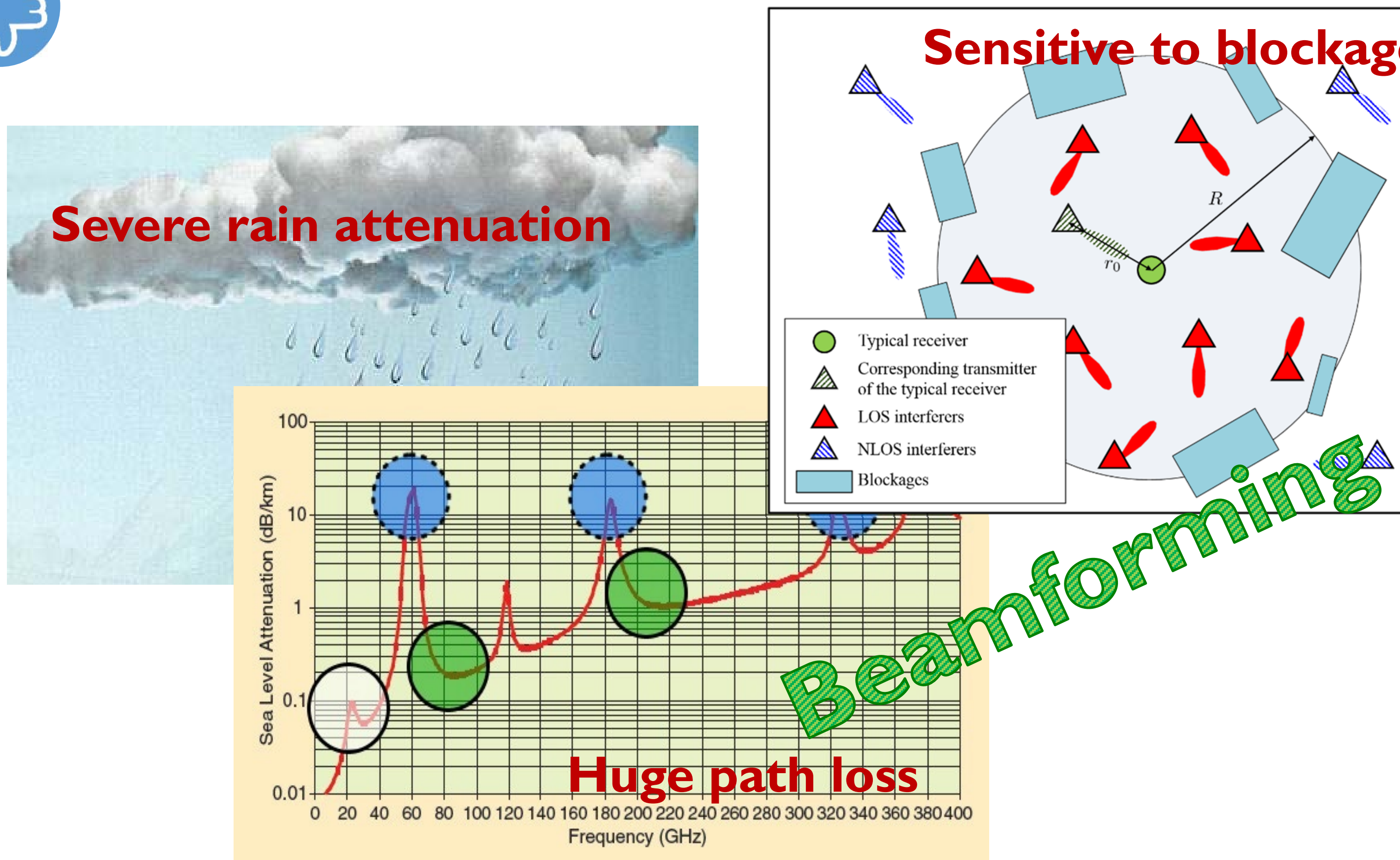


[U.S. Frequency Allocation Chart as of October 2011]

- mm-wave bands: Less congested, more bandwidth

Hybrid Precoding for mm-Wave MIMO Systems

Background and Motivation



[G. R. MacCartney *et al.*, 2013]

Hybrid Precoding for mm-Wave MIMO Systems
Background and Motivation

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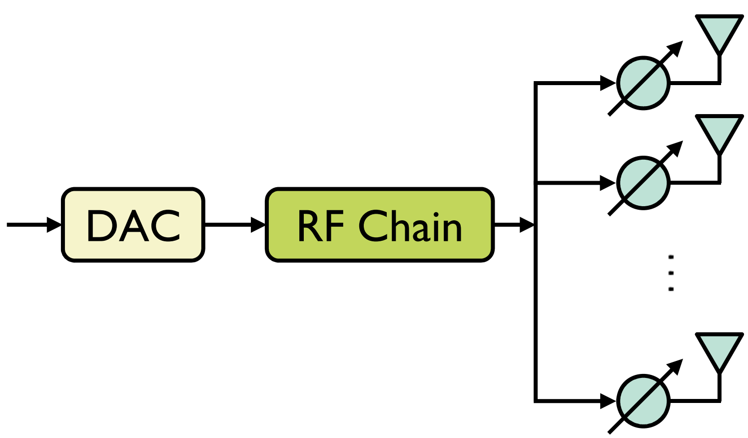
- Large-scale antennas can be patched together
  - Large antenna gain to compensate the path loss
- Conventional Approach: **Analog beamforming**
  - State-of-art in mm-wave WiGig systems [E. Perahia *et al.*, 2010]



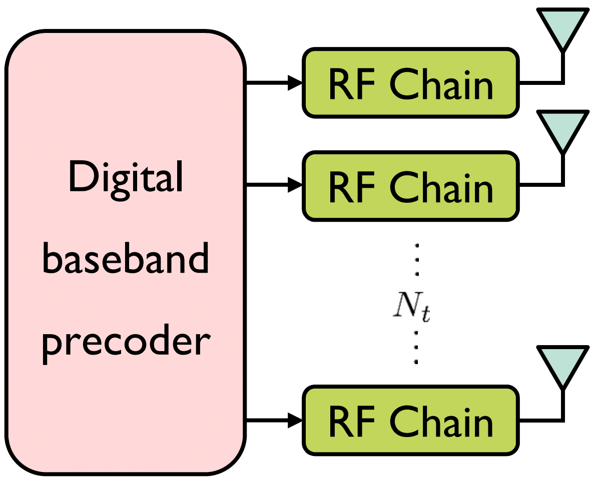
- ✔ Phased array
- ✘ Spatial multiplexing
- ✘ SDMA

Hybrid Precoding for mm-Wave MIMO Systems

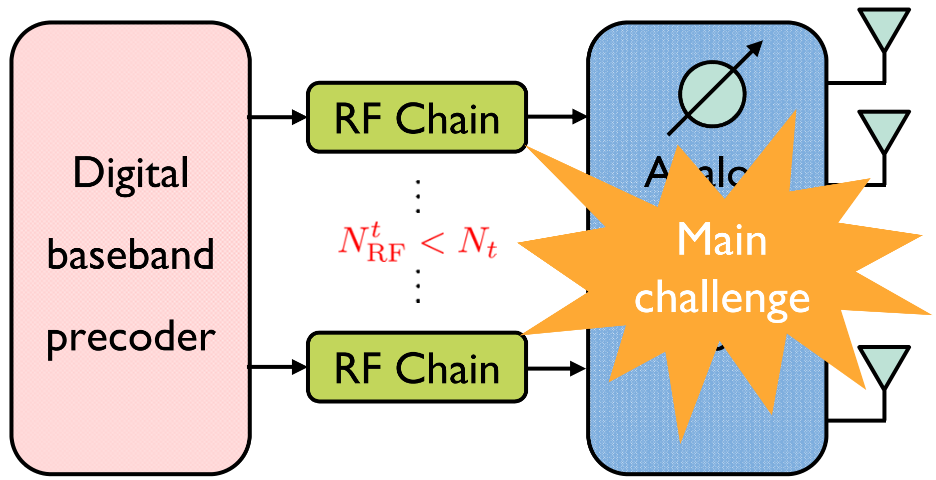
Introduction to Hybrid Precoding

- New transceiver architecture

Sub-6 GHz systems: fully digital precoder



mm-wave systems: hybrid precoder

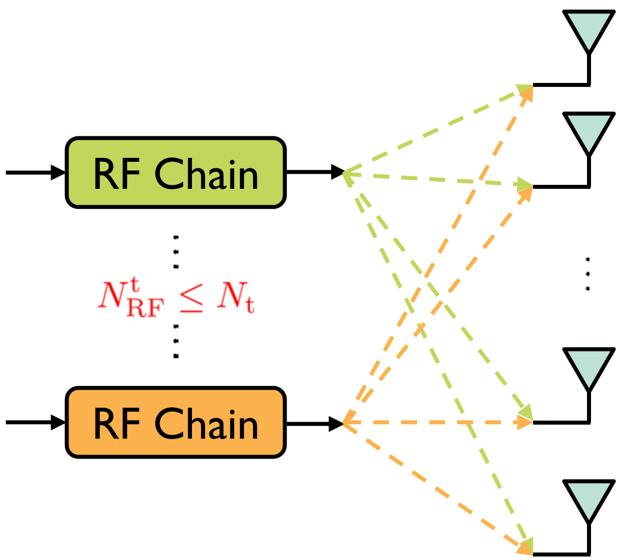


Q1: Can it approach the performance of the fully digital precoding?

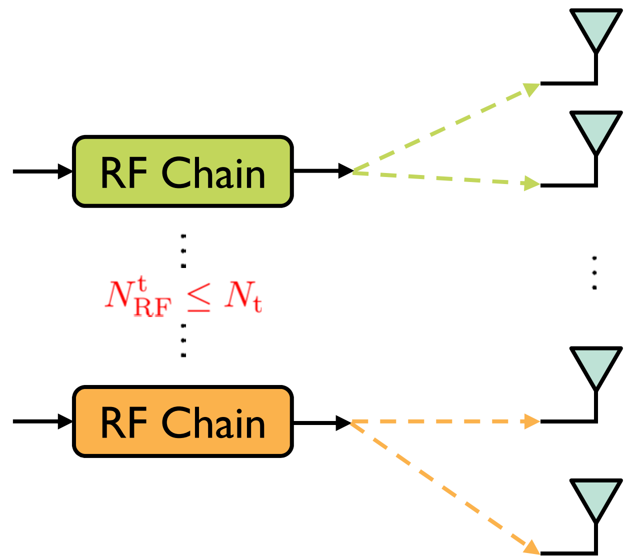
Q2: How many RF chains are needed?

- Key differentiating component
  - Mapping from RF chains to antennas

- Analog RF precoder structure
  - Signal flow determines the "mapping strategy"



Fully-connected mapping



Partially-connected mapping

Q3: How to connect RF chains and antennas?

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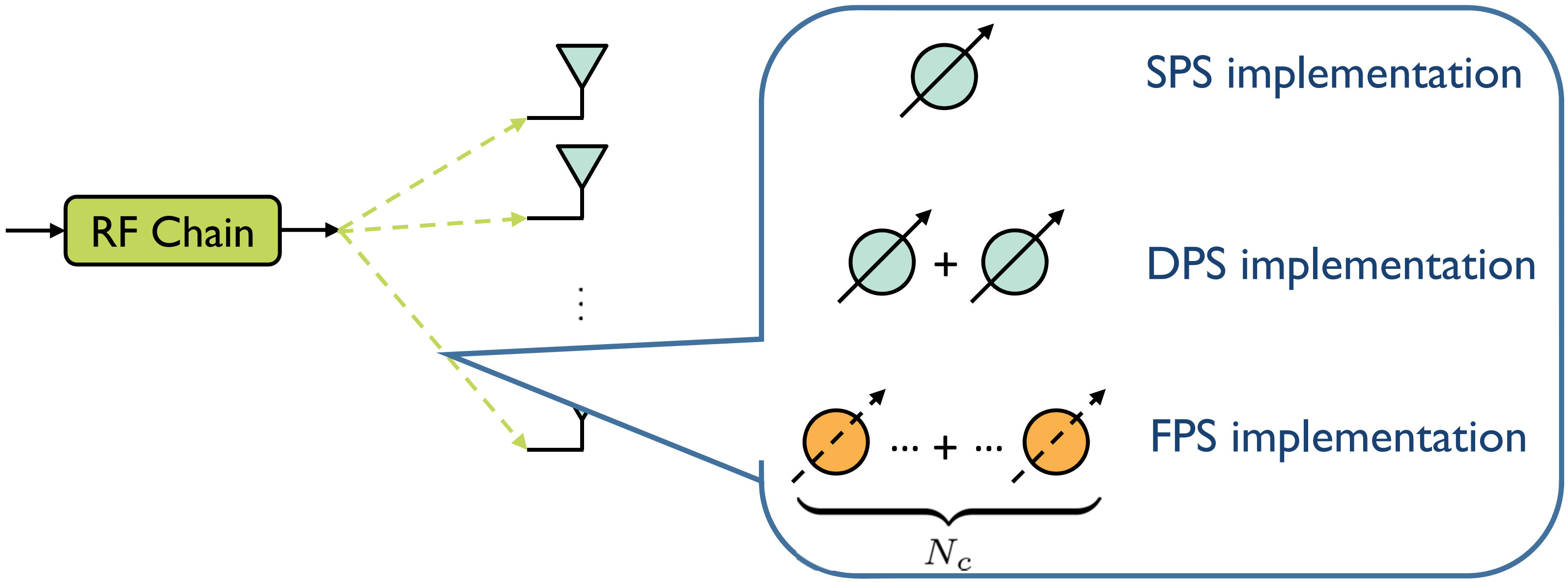
- Analog RF precoder structure (cont'd)
  - Adopted hardware determines the "implementation"
  - For each connected signal flow



Q4: How many phase shifters are needed?

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Introduction to Hybrid Precoding

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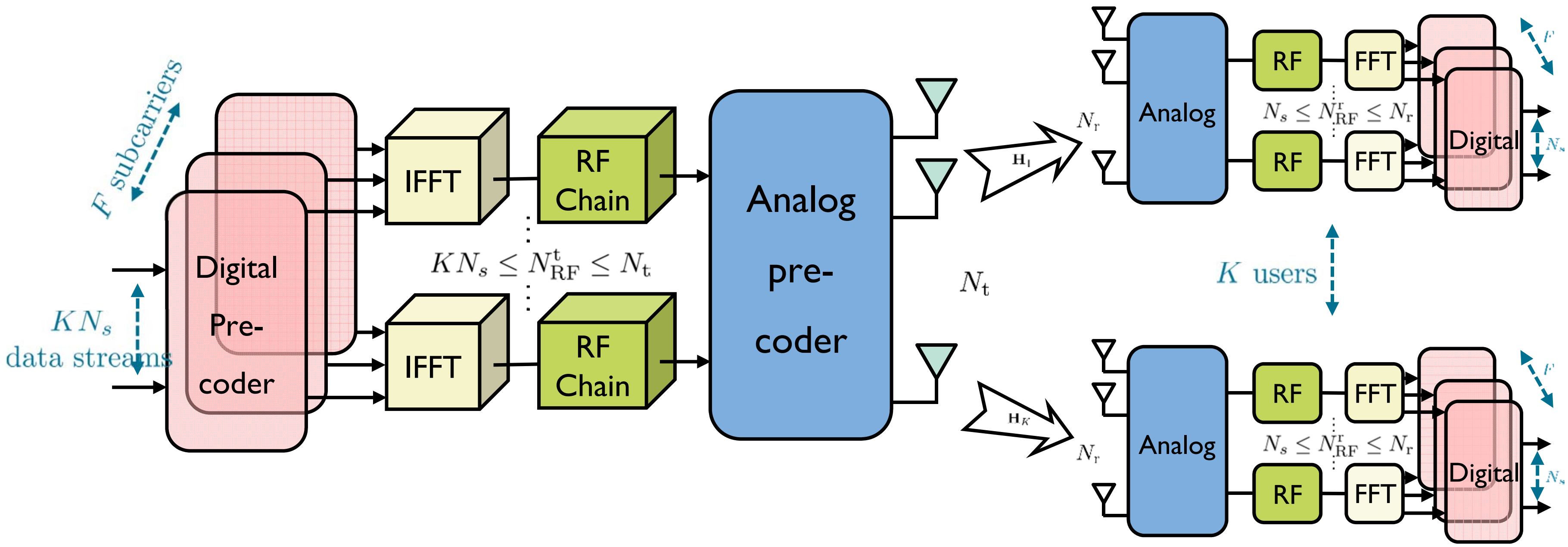
❖ General multiuser multicarrier systems



➢ One single digital precoder $\mathbf{F}_{\mathrm{BB}k,f}$ for each user on each subcarrier

➢ Analog precoder $\mathbf{F}_{\mathrm{RF}}$ is shared by all the users and subcarriers

Problem formulation

- Minimize the Euclidean distance between the hybrid precoders and the fully digital precoder [O. El Ayach *et al.*, 2014]

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \|\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \leq P_{\max} \\ & \mathbf{F}_{\mathrm{RF}} \in \mathcal{A}_x \end{aligned}$$

Power constraint

Difficulty

- $\mathcal{A}_x$ varies according to different mappings and implementations

$$\mathbf{F}_{\mathrm{opt}} = \left[\mathbf{F}_{\mathrm{opt}\,1,1}, \cdots, \mathbf{F}_{\mathrm{opt}\,k,f}, \cdots, \mathbf{F}_{\mathrm{opt}\,K,F}\right] \in N_{\mathrm{t}} \times KN_sF$$

$$\mathbf{F}_{\mathrm{BB}} = \left[\mathbf{F}_{\mathrm{BB}1,1}, \cdots, \mathbf{F}_{\mathrm{BB}k,f}, \cdots, \mathbf{F}_{\mathrm{BB}K,F}\right] \in N_{\mathrm{RF}}^{\mathrm{t}} \times KN_sF$$

Q4: How to efficiently design hybrid precoding algorithms?

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- Existing works
  - Most focused on the SPS implementation
    - Orthogonal Matching Pursuit (OMP) [O. El Ayach *et al.*, 2014] [T. E. Bogale *et al.*, 2014]
      - A candidate set for $\mathbf{F}_{\mathrm{RF}}$
      - Array response vectors or DFT matrix
    - Channel phase extraction [L. Liang *et al.*, 2014]
    - Successive interference cancellation (SIC) [X. Gao *et al.*, 2016]
  - How to achieve the fully digital precoder [E. Zhang *et al.*, 2014] [T. E. Bogale *et al.*, 2016]
    - Large numbers of RF chains and PSs needed

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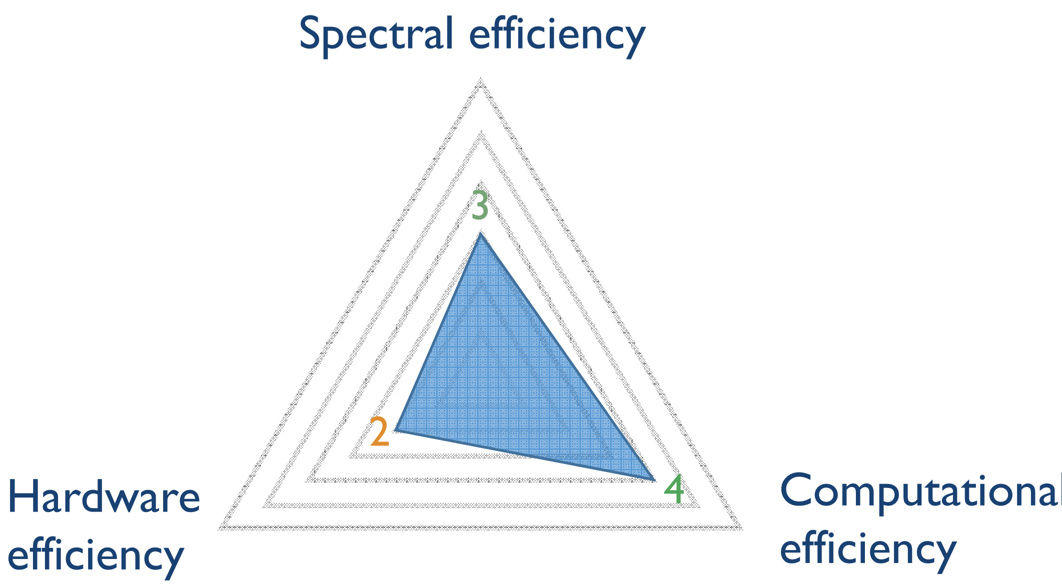
Introduction to Hybrid Precoding

- Performance metrics
  - “Scoring triangle”



Baseline: SPS fully-connected with OMP

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Key Questions

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- **Q1**: Can hybrid precoder provide performance close to the fully digital one?
- **Q2**: How many RF chains are needed?
- **Q3**: How to connect the RF chains and antennas?
- **Q4**: How many phase shifters are needed?
- **Q5**: How to efficiently design hybrid precoding algorithms?

Hybrid Precoding for mm-Wave MIMO Systems

Key Design Aspects

- **Hardware complexity (# hardware components)**
- **Computational efficiency of precoding algorithms**
- **Achievable spectrum efficiency**

Hybrid Precoding for mm-Wave MIMO Systems

Algorithm Design & Hardware Implementation I: SPS

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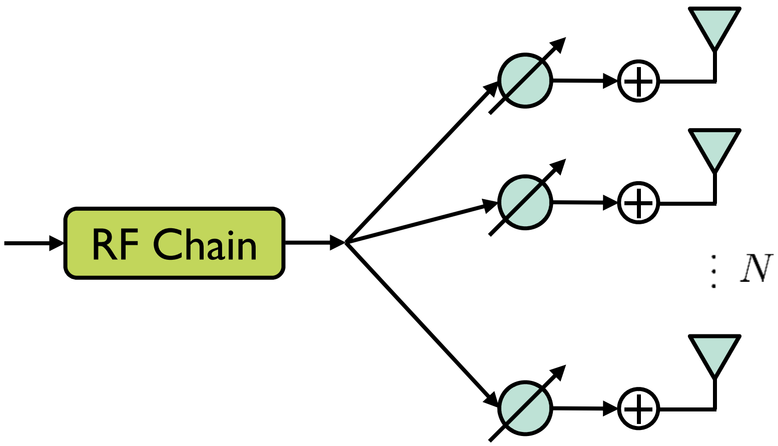
- Single phase shifter (SPS) implementation



$$N = \begin{cases} N_{\mathrm{t}} & \text{fully-connected} \\ N_{\mathrm{t}}/N_{\mathrm{RF}}^{\mathrm{t}} & \text{partially-connected} \end{cases}$$

- Fully digital achieving condition:

$$N_{\mathrm{RF}}^{\mathrm{t}} = 2KN_s,\ N_{\mathrm{RF}}^{\mathrm{r}} = 2N_s$$

Q: Can we further reduce the number of RF chains?

- Start from single-user systems
  - Alternating minimization

[Ref] X. Yu, J.-C. Shen, J. Zhang, and K. B. Letaief, "Alternating minimization algorithms for hybrid precoding in millimeter wave MIMO systems," *IEEE J. Sel. Topics Signal Process.*, vol. 10, no. 3, pp. 485-500, Apr. 2016.

Hybrid Precoding for mm-Wave MIMO Systems

Algorithm Design & Hardware Implementation I: SPS

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- Fully-connected mapping

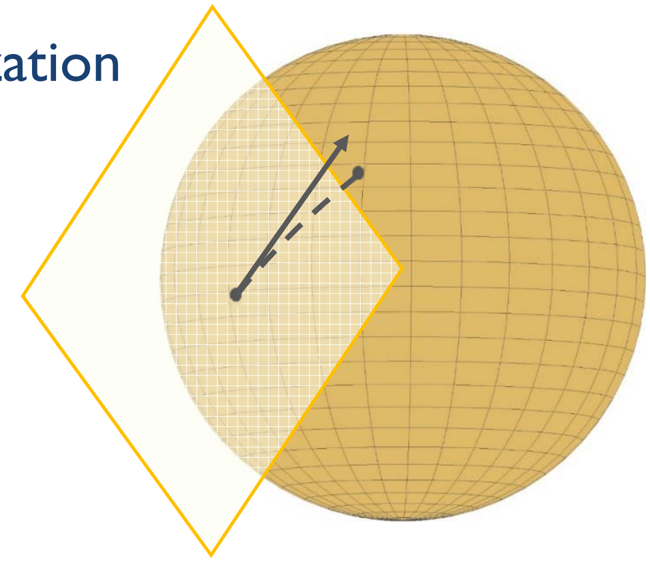
$$\underset{\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

- Digital precoder: $\mathbf{F}_{\mathrm{BB}} = \mathbf{F}_{\mathrm{RF}}^{\dagger}\mathbf{F}_{\mathrm{opt}}$
- Difficulty: Analog precoder design with the unit modulus constraints

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & |(\mathbf{F}_{\mathrm{RF}})_{i,j}| = 1, \forall i, j. \end{aligned}$$

- The vector $\mathbf{x} = \mathrm{vec}(\mathbf{F}_{\mathrm{RF}})$ forms a complex circle manifold

$$\mathcal{M}^m = \{\mathbf{x} \in \mathbb{C}^m : |\mathbf{x}_1| = |\mathbf{x}_2| = \cdots = |\mathbf{x}_m| = 1\}, \quad m = N_{\mathrm{t}}N_{\mathrm{RF}}^{\mathrm{t}}.$$



- 3 key elements in manifold optimization
  - Tangent space:

$$T_{\mathbf{x}}\mathcal{M}^m = \{\mathbf{y} \in \mathbb{C}^m : \Re\{\mathbf{y} \circ \mathbf{x}^*\} = \mathbf{0}_m\}$$

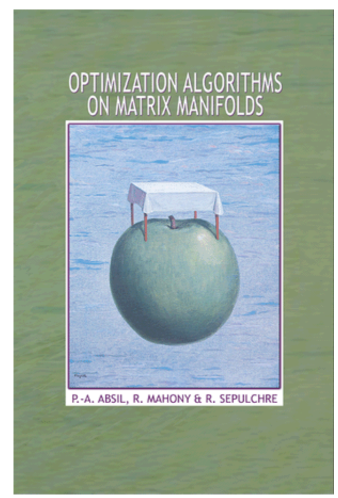
  - Riemannian gradient

$$\begin{aligned}\operatorname{grad} f(\mathbf{x}) &= \operatorname{Proj}_{\mathbf{x}} \nabla f(\mathbf{x}) \\ &= \nabla f(\mathbf{x}) - \Re\{\operatorname{diag}[\nabla f(\mathbf{x}) \circ \mathbf{x}^*]\}\mathbf{x}\end{aligned}$$

$$\nabla f(\mathbf{x}) = -2(\mathbf{F}_{\mathrm{BB}}^* \otimes \mathbf{I}_{N_t})\left[\operatorname{vec}(\mathbf{F}_{\mathrm{opt}}) - (\mathbf{F}_{\mathrm{BB}}^T \otimes \mathbf{I}_{N_t})\mathbf{x}\right]$$

  - Retraction: $\operatorname{Retr}_{\mathbf{x}} : T_{\mathbf{x}}\mathcal{M}^m \to \mathcal{M}^m :$

$$\alpha\mathbf{d} \mapsto \operatorname{Retr}_{\mathbf{x}}(\alpha\mathbf{d}) = \operatorname{vec}\left[\frac{(\mathbf{x} + \alpha\mathbf{d})_i}{|(\mathbf{x} + \alpha\mathbf{d})_i|}\right]$$



- Conjugate gradient algorithm
  - Local optimum guaranteed

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Partially-connected mapping

- Block diagonal structure of $\mathbf{F}_{\mathrm{RF}}$

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\mathrm{RF}}^t} \end{bmatrix}$$

$$\mathbf{p}_i = \left[\exp\left(j\theta_{(i-1)\frac{N_t}{N_{\mathrm{RF}}^t}+1} \right), \cdots, \exp\left(j\theta_{i\frac{N_t}{N_{\mathrm{RF}}^t}} \right) \right]^T$$

phase shifters connected to the i-th RF chain

- Problem decoupled for each RF chain

- Closed-form solution for $\mathbf{F}_{\mathrm{RF}}$

$$\arg\left\{(\mathbf{F}_{\mathrm{RF}})_{i,l}\right\} = \arg\left\{ (\mathbf{F}_{\mathrm{opt}})_{i,:} (\mathbf{F}_{\mathrm{BB}})_{l,:}{}^H \right\}, \quad 1 \le i \le N_t, l = \left\lceil i \frac{N_{\mathrm{RF}}^t}{N_t} \right\rceil$$

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation I: SPS

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- Partially-connected mapping (cont'd)
  - Optimization of $\mathbf{F}_{\mathrm{BB}}$

$$
\begin{aligned}
\underset{\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\
\text{subject to} \quad & \|\mathbf{F}_{\mathrm{BB}}\|_F^2 = \frac{N_{\mathrm{RF}}^t N_s}{N_t}.
\end{aligned}
$$

  - Reformulate as a non-convex QCQP problem

$$
\begin{aligned}
\underset{\mathbf{Y} \in \mathbb{H}^n}{\text{minimize}} \quad & \mathrm{Tr}(\mathbf{C}\mathbf{Y}) \\
\text{subject to} \quad & \begin{cases} \mathrm{Tr}(\mathbf{A}_1\mathbf{Y}) = \frac{N_{\mathrm{RF}}^t N_s}{N_t} \\ \mathrm{Tr}(\mathbf{A}_2\mathbf{Y}) = 1 \\ \mathbf{Y} \succeq 0, \ \cancel{\mathrm{rank}(\mathbf{Y}) = 1} \end{cases}
\end{aligned}
$$

  - SDR is tight so globally optimal solution is obtained [Z.-Q. Luo *et al.*, 2010]
- Converge to a local optimum

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation I: SPS

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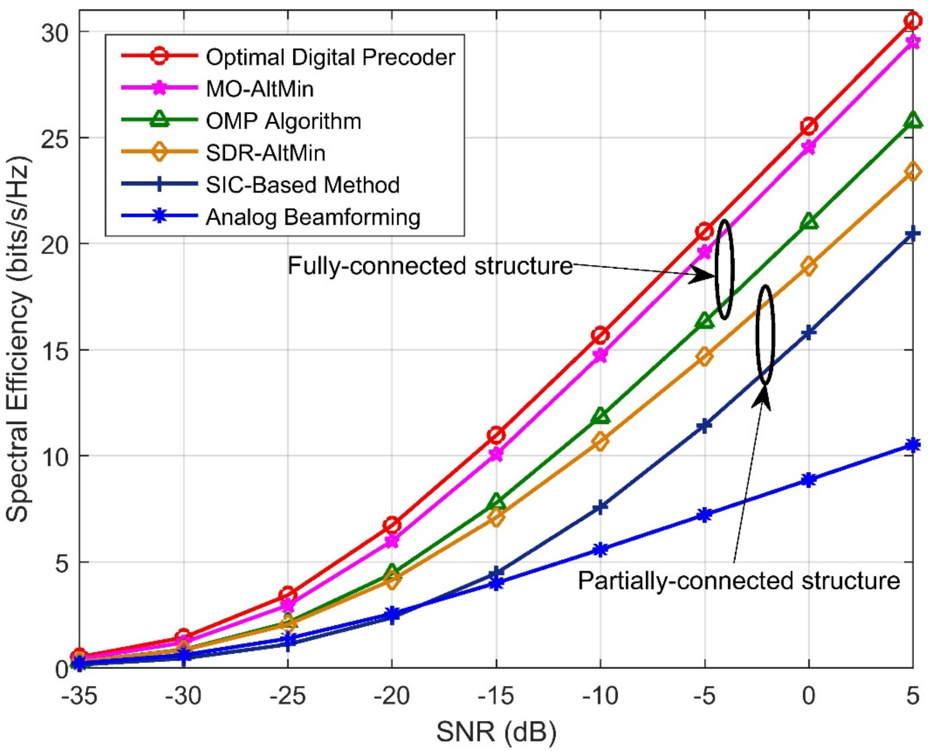
Simulation results

$$N_t = 144,\ N_r = 36,\ N_{RF}^t = N_{RF}^r = N_s = 3$$



Effectiveness of the proposed AltMin algorithms

The fully-connected mapping can easily approach the performance of the fully digital precoding

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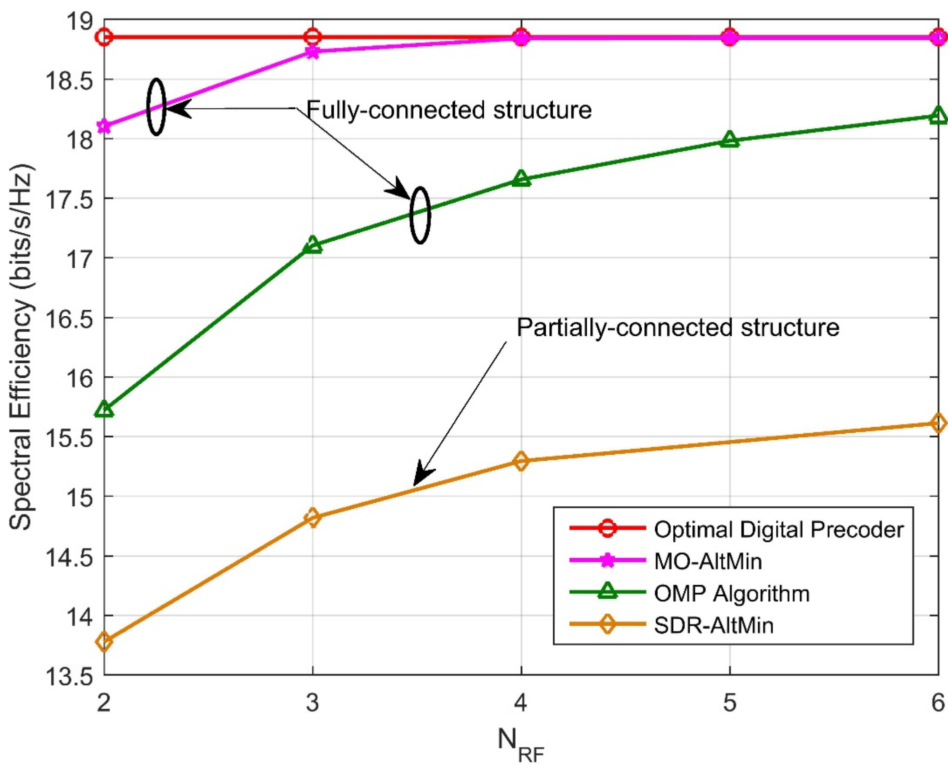
Simulation results

$$N_{\mathrm{t}} = 144,\ N_{\mathrm{r}} = 36,\ N_{\mathrm{RF}}^{\mathrm{t}} = N_{\mathrm{RF}}^{\mathrm{r}} = N_{\mathrm{RF}},\ N_s = 2,\ \mathrm{SNR} = 0\,\mathrm{dB}$$



~N_s RF chains are enough for the fully-connected mapping

Employing fewer PSs, the partially-connected mapping needs more RF chains

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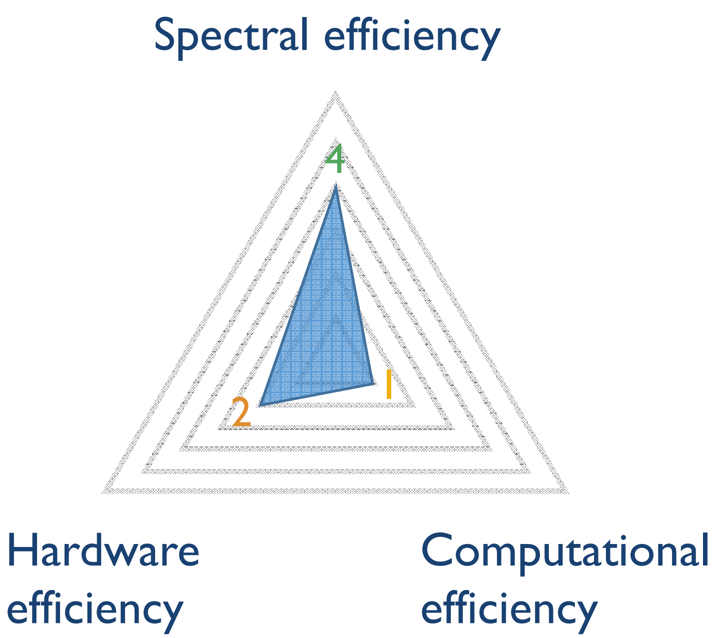
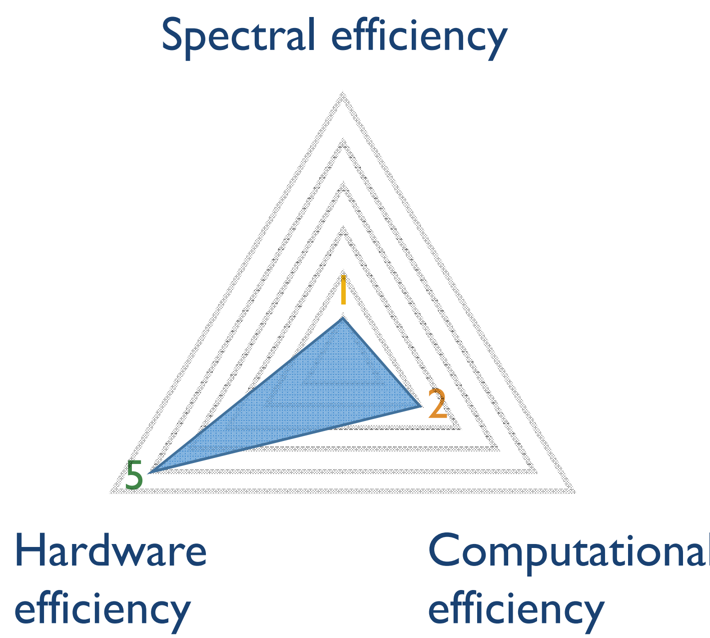
❖ Conclusion



SPS fully-connected



SPS partially-connected

❖ **Limitation**: computational efficiency not good, thus difficult to extend to MU multicarrier settings

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation II: DPS

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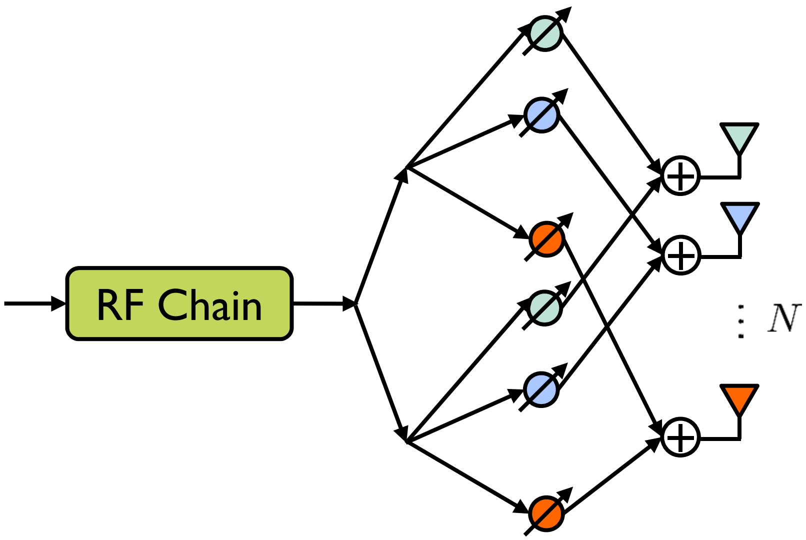
❖ Double phase shifter (DPS) implementation



$$N = \begin{cases} N_{\rm t} & \text{fully-connected} \\ N_{\rm t}/N_{\rm RF}^{\rm t} & \text{partially-connected} \end{cases}$$

Q: What is the benefit?

➢ Sum of two phase shifters $|e^{\jmath\theta_1} + e^{\jmath\theta_2}| \leq 2$

[Ref] X. Yu, J. Zhang, and K. B. Letaief, "Doubling phase shifters for efficient hybrid precoder design in millimeter-wave communication systems," submitted to *IEEE Trans. Wireless Commun.*, Mar. 2017.

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- Fully-connected mapping
  - RF-only precoding

$$\begin{aligned} &\underset{\mathbf{F}_{\mathrm{RF}}}{\text{minimize}} && \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ &\text{subject to} && |(\mathbf{F}_{\mathrm{RF}})_{i,j}| \le 2 \end{aligned} \Longleftrightarrow \underset{\mathbf{x}}{\text{minimize}} \quad \frac{1}{2}\|\mathbf{A}\mathbf{x} - \mathbf{b}\|_2^2 + 2\|\mathbf{x}\|_1$$

LASSO

  - Closed-form solution for semi-unitary codebooks $\mathbf{F}_{\mathrm{BB}}\mathbf{F}_{\mathrm{BB}}^H = \mathbf{I}_{N_{\mathrm{RF}}^{\mathrm{t}}}$

$$\mathbf{F}_{\mathrm{RF}}^\star = \mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H - \exp\left\{j\angle\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H\right)\right\} \circ \left(\left|\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{BB}}^H\right| - 2\right)^+ .$$

  - Hybrid precoding

$$\underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

Redundant

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Fully-connected mapping (cont'd)

- Optimality in single-carrier systems

$$\mathbf{F}_{\mathrm{opt}} = \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}} \text{ with } \underline{N_{\mathrm{RF}}^{\mathrm{t}} = KN_s} \text{ and } \underline{N_{\mathrm{RF}}^{\mathrm{r}} = N_s} \text{ when } F = 1$$

Minimum number of RF chains

- Reduced the number of RF chains by half required for achieving the fully digital precoding

- Multi-carrier systems

$$\underset{\mathbf{F}_{\mathrm{RF}},\mathbf{F}_{\mathrm{BB}}}{\mathrm{minimize}} \quad \|\mathbf{F}_{\mathrm{opt}} - \mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}}\|_F^2$$

- Low-rank matrix approximation: SVD
  - **Optimal solution**

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- Partially-connected mapping
  - Block diagonal structure

$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{p}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{p}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{p}_{N_{\mathrm{RF}}^{\mathrm{t}}} \end{bmatrix}$$

Not much performance gain obtained by the DFS implementation in the partially-connected mapping

$$\mathbf{p}_j = \left[a_{(j-1)\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}}+1}, \cdots, a_{j\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}}} \right]^T$$

  - Decoupled for each RF chain

$$\mathcal{P}_j : \quad \underset{\{a_i\}, \mathbf{x}_j}{\text{minimize}} \sum_{i \in \mathcal{F}_j} \|\mathbf{y}_i - a_i \mathbf{x}_j\|_2^2 ,$$

$$\mathcal{F}_j = \left\{ i \in \mathbb{Z} \,\middle|\, (j-1)\frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}} + 1 \leq i \leq j \frac{N_{\mathrm{t}}}{N_{\mathrm{RF}}^{\mathrm{t}}} \right\}, \; \mathbf{y}_i = \mathbf{F}_{\mathrm{opt}}^T(i,:), \text{ and } \mathbf{x}_j = \mathbf{F}_{\mathrm{BB}}^T(j,:)$$

  - Eigenvalue problem

$$\mathbf{x}_j^\star = \boldsymbol{\lambda}_1 \left(\sum_{i \in \mathcal{F}_j} \mathbf{y}_i \mathbf{y}_i^H \right), \quad a_i^\star = \frac{\mathbf{x}_j^H \mathbf{y}_i}{\|\mathbf{x}_j\|_2^2}$$

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Partially-connected mapping (cont'd)

- Dynamic mapping

  Adaptively separate all the antennas into N_{RF} groups

- Problem formulation

$$\begin{aligned} \underset{\{\mathcal{D}_j\}_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}}}{\text{maximize}} \quad & \sum_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}} \lambda_1 \left(\sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right) \\ \text{subject to} \quad & \begin{cases} \cup_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}} \mathcal{D}_j = \{1, \cdots, N_{\mathrm{t}}\} \\ \mathcal{D}_j \cap \mathcal{D}_k = \emptyset, \quad \forall j \neq k \end{cases} \end{aligned}
\quad \Rightarrow \quad
\begin{aligned} \underset{\{\mathcal{D}_j, \mathbf{x}_j\}_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}}}{\text{maximize}} \quad & \sum_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}} \frac{\mathbf{x}_j^H \left(\sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right) \mathbf{x}_j}{\mathbf{x}_j \mathbf{x}_j^H} \\ \text{subject to} \quad & \begin{cases} \cup_{j=1}^{N_{\mathrm{RF}}^{\mathrm{t}}} \mathcal{D}_j = \{1, \cdots, N_{\mathrm{t}}\} \\ \mathcal{D}_j \cap \mathcal{D}_k = \emptyset, \quad \forall j \neq k \end{cases} \end{aligned}$$

- Modified K-means algorithm
  - Centroid: $\mathbf{x}_j^\star = \boldsymbol{\lambda}_1 \left(\sum_{i \in \mathcal{D}_j} \mathbf{y}_i \mathbf{y}_i^H \right)$
  - Clustering: $j^\star = \arg\max_j \quad \left| \mathbf{y}_i^H \mathbf{x}_j \right|^2$

- Converge to a local optimum

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- Inter-user interference
  - Approximating the fully digital precoder leads to a near-optimal performance in single-user single-carrier, single-user multicarrier, and multiuser single-carrier mmWave MIMO systems.
  - Inter-user interference will be more prominent in the multicarrier system as the analog precoder is shared by a large number of subcarriers
  - Cascade an additional block diagonalization (BD) precoder
    - Effective channel: $\hat{\mathbf{H}}_{k,f} = \mathbf{W}_{\mathrm{BB}k,f}^{H}\mathbf{W}_{\mathrm{RF}k}^{H}\mathbf{H}_{k,f}\mathbf{F}_{\mathrm{RF}}\mathbf{F}_{\mathrm{BB}f}$
    - BD: $\hat{\mathbf{H}}_{j,f}\mathbf{F}_{\mathrm{BD}k,f} = \mathbf{0}, \quad k \neq j$

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Algorithm Design & Hardware Implementation II: DPS

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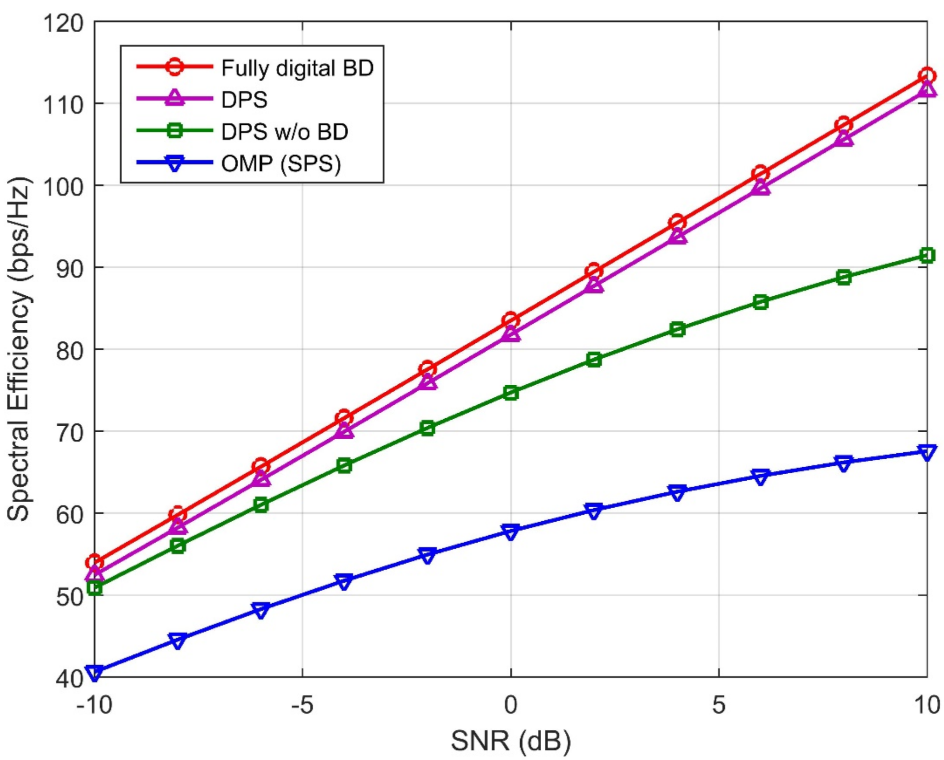
❖ Simulation results (Fully-connected)

$N_\mathrm{t} = 256,\ N_\mathrm{r} = 16,\ K = 3,\ F = 128,\ N_s = 3,\ N_\mathrm{RF}^\mathrm{t} = 9$, and $N_\mathrm{RF}^\mathrm{r} = 3$



Achieve near-optimal spectral efficiency and optimal multiplexing gain with low-complexity algorithms

Effectiveness of the proposed DPS implementation and cascaded BD precoder

Hybrid Precoding for mm-Wave MIMO Systems

Algorithm Design & Hardware Implementation II: DPS

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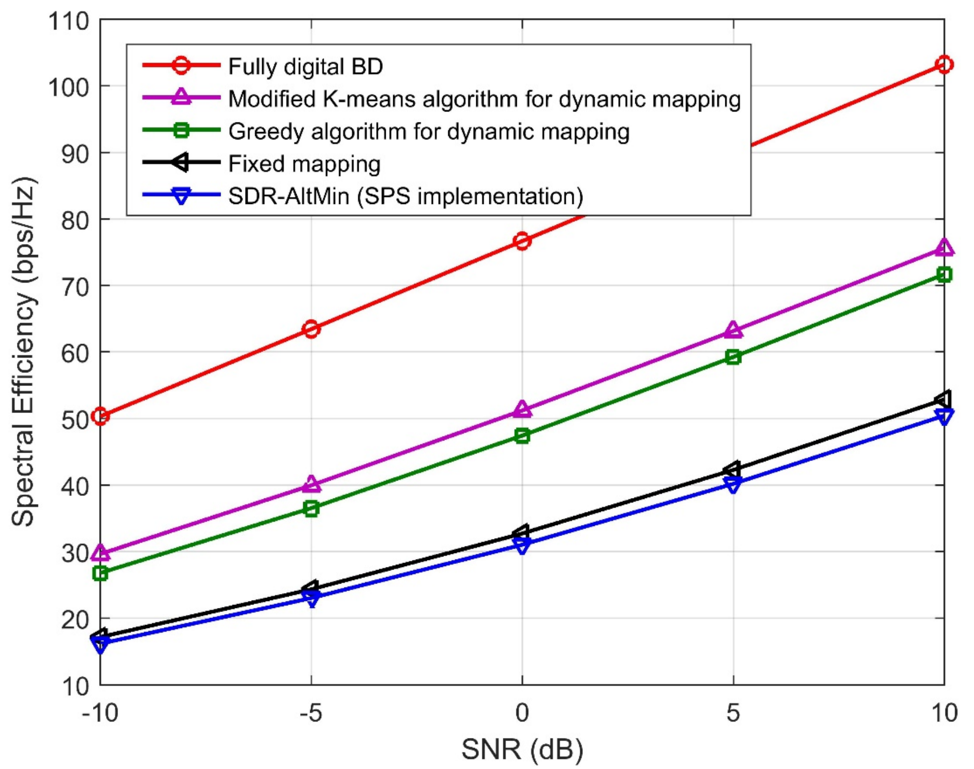
Simulation results (Partially-connected)

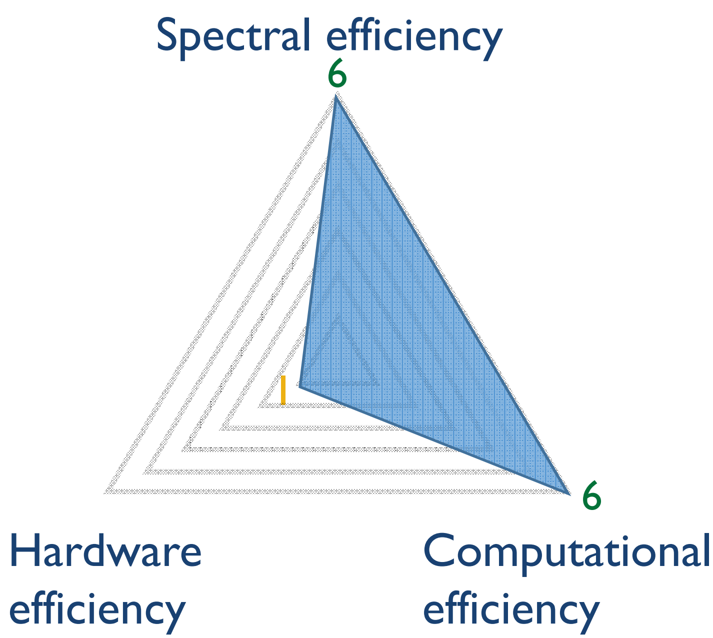
$N_t = 256,\ N_r = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{RF}^t = 8$, and $N_{RF}^r = 2$



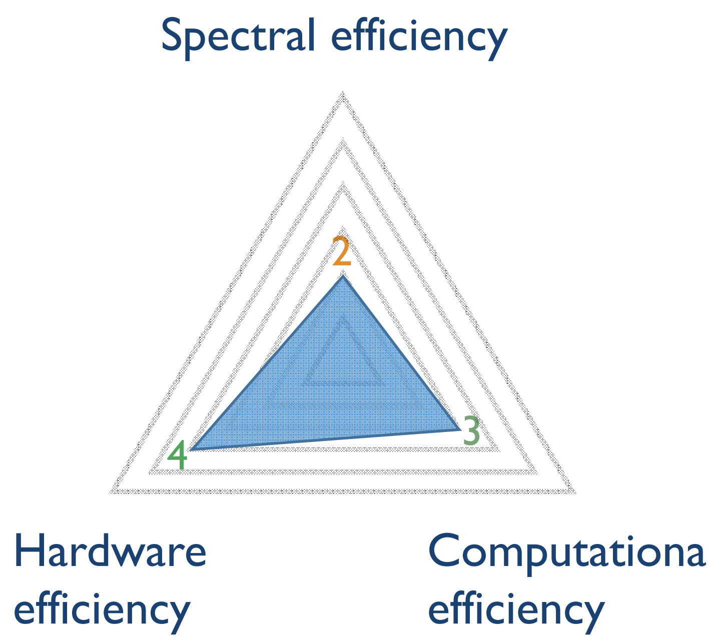
Simply doubling PSs in the partially-connected mapping is far from satisfactory

Superiority of the modified K-means algorithm with lower complexity than the greedy one

❖ Conclusion



DPS fully-connected



DPS partially-connected

❖ **Limitations**

- ❖ Too many phase shifters needed
- ❖ Adaptive high-resolution phases needed

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation III: FPS

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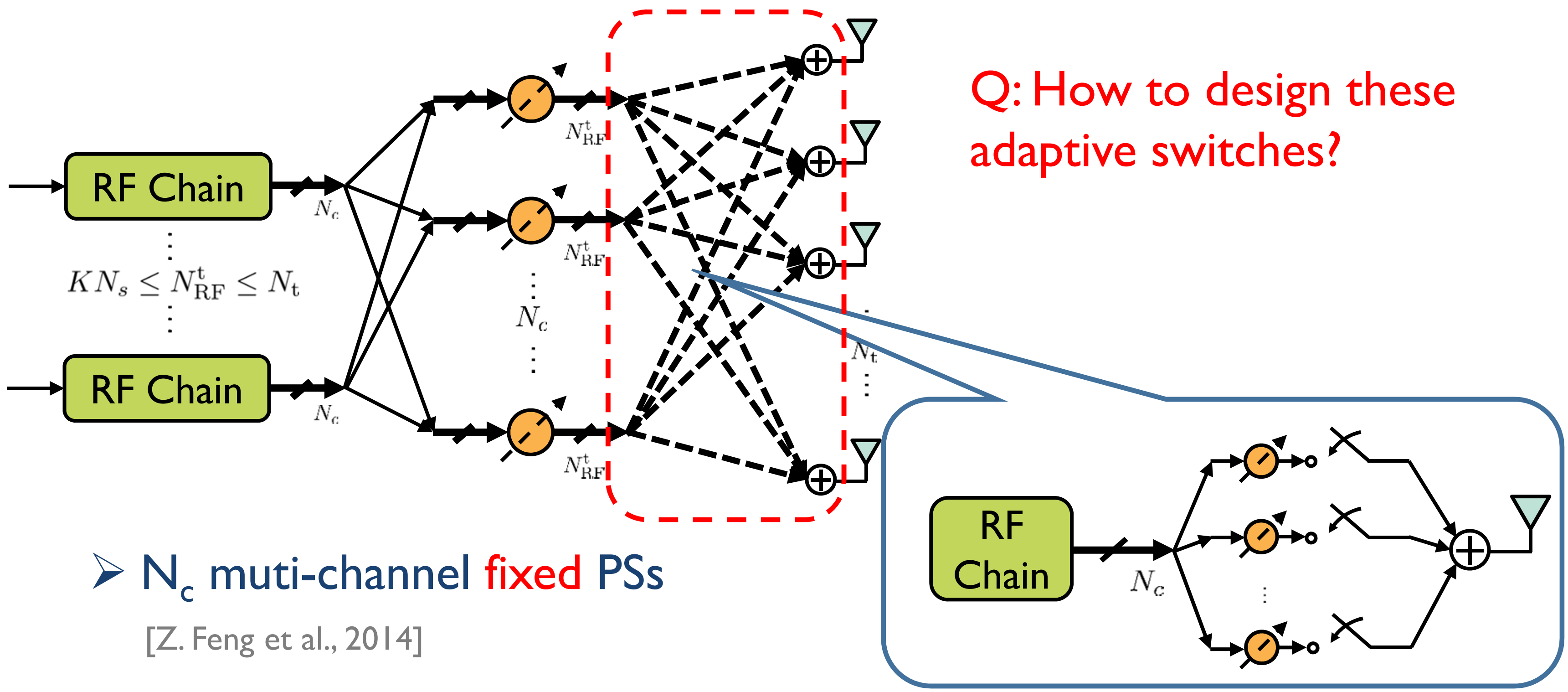
- Fixed phase shifter (FPS) implementation



- N_c muti-channel fixed PSs

[Z. Feng et al., 2014]

[Ref] X. Yu, J. Zhang, and K. B. Letaief, "Hybrid precoding in millimeter wave systems: How many phase shifters are needed?" accepted to *IEEE Global Commun. Conf. (GLOBECOM), Singapore, Dec. 2017.*

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation III: FPS

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- Problem formulation

$$\begin{aligned} \underset{\mathbf{S},\mathbf{F}_{\mathrm{BB}}}{\text{minimize}} \quad & \|\mathbf{F}_{\mathrm{opt}} - \mathbf{S}\mathbf{C}\mathbf{F}_{\mathrm{BB}}\|_F^2 \\ \text{subject to} \quad & \mathbf{S} \in \mathcal{B} \end{aligned}$$

  - FPS matrix $\mathbf{C} = \mathrm{diag}\left(\underbrace{\mathbf{c}, \mathbf{c}, \cdots, \mathbf{c}}_{N_{\mathrm{RF}}^{\mathrm{t}}}\right), \quad \mathbf{c} = \frac{1}{\sqrt{N_c}}\left[e^{\jmath\theta_1}, e^{\jmath\theta_2}, \cdots, e^{\jmath\theta_{N_c}}\right]^T$
  - Switch matrix $\mathbf{S} \in \{0,1\}^{N_{\mathrm{t}} \times N_c N_{\mathrm{RF}}^{\mathrm{t}}}$

- An objective upper bound enables a low-complexity algorithm
  - Enforce a semi-orthogonal constraint of $\mathbf{F}_{\mathrm{BB}}$ [X.Yu et al., 2016]

$$\mathbf{F}_{\mathrm{BB}}^H \mathbf{F}_{\mathrm{BB}} = \alpha^2 \mathbf{F}_{\mathrm{DD}}^H \mathbf{F}_{\mathrm{DD}} = \alpha^2 \mathbf{I}_{KN_s}$$

$$\|\mathbf{F}_{\mathrm{opt}} - \mathbf{S}\mathbf{C}\mathbf{F}_{\mathrm{BB}}\|_F^2 \leq \|\mathbf{F}_{\mathrm{opt}}\|_F^2 - 2\alpha \Re \,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}} \mathbf{F}_{\mathrm{opt}}^H \mathbf{S}\mathbf{C}\right) + \alpha^2 \|\mathbf{S}\|_F^2$$

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation III: FPS

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- Alternating minimization
  - Digital precoder

$$\begin{aligned} \underset{\mathbf{F}_{\mathrm{DD}}}{\text{maximize}} \quad & \Re\,\mathrm{Tr}\left(\mathbf{F}_{\mathrm{DD}}\mathbf{F}_{\mathrm{opt}}^{H}\mathbf{S}\mathbf{C}\right) \\ \text{subject to} \quad & \mathbf{F}_{\mathrm{DD}}^{H}\mathbf{F}_{\mathrm{DD}} = \mathbf{I}_{KN_s} \end{aligned}$$

  - Semi-orthogonal Procrustes solution $\mathbf{F}_{\mathrm{DD}} = \mathbf{V}_1\mathbf{U}^H$

  - Switch matrix optimization

$$\begin{aligned} \underset{\alpha,\mathbf{S}}{\text{minimize}} \quad & \left\|\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) - \alpha\mathbf{S}\right\|_F^2 \\ \text{subject to} \quad & \mathbf{S} \in \mathcal{B} \end{aligned}$$

  - Once α is optimized, the optimal **S** is determined correspondingly

$$\mathbf{S}^{\star} = \begin{cases} \mathcal{I}\left\{\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) > \frac{\alpha}{2}\mathbf{1}_{N_{\mathrm{t}}\times N_c N_{\mathrm{RF}}^{\mathrm{t}}}\right\} & \alpha > 0 \\ \mathcal{I}\left\{\Re\left(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^{H}\mathbf{C}^{H}\right) < \frac{\alpha}{2}\mathbf{1}_{N_{\mathrm{t}}\times N_c N_{\mathrm{RF}}^{\mathrm{t}}}\right\} & \alpha < 0 \end{cases}$$

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- Alternating minimization (cont'd)
  - Optimization of α

$$\alpha^\star = \arg \min_{\{\tilde{x}_i, \bar{x}_i\}_{i=1}^n} \{f(\tilde{x}_i), f(\bar{x}_i)\}$$

$$\tilde{\mathbf{x}} = \mathrm{sort}(\mathrm{vec}(\mathbf{F}_{\mathrm{opt}}\mathbf{F}_{\mathrm{DD}}^H\mathbf{C}^H)) \qquad \bar{x}_i \triangleq \begin{cases} \frac{\sum_{j=1}^{i}\tilde{x}_j}{i} & \alpha < 0 \text{ and } \frac{\sum_{j=1}^{i}\tilde{x}_j}{i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ \frac{\sum_{j=i+1}^{n}\tilde{x}_j}{n-i} & \alpha > 0 \text{ and } \frac{\sum_{j=i+1}^{n}\tilde{x}_j}{n-i} \in [2\tilde{x}_i, 2\tilde{x}_{i+1}] \\ +\infty & \text{otherwiese} \end{cases}$$

  - Search dimension: $2N_{\mathrm{t}}N_{\mathrm{RF}}^{\mathrm{t}}N_c$
  - Optimal point can only be finite $\bar{x}_i$

$$\alpha^\star = \arg \min_{\bar{x}_i \in \mathcal{X}} \quad f(\bar{x}_i)$$

  - Search dimension: $|\mathcal{X}| \ll 2N_{\mathrm{t}}N_{\mathrm{RF}}^{\mathrm{t}}N_c$
- Converge to a local optimum

Q: How to reduce # phase shifters?

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation III: FPS

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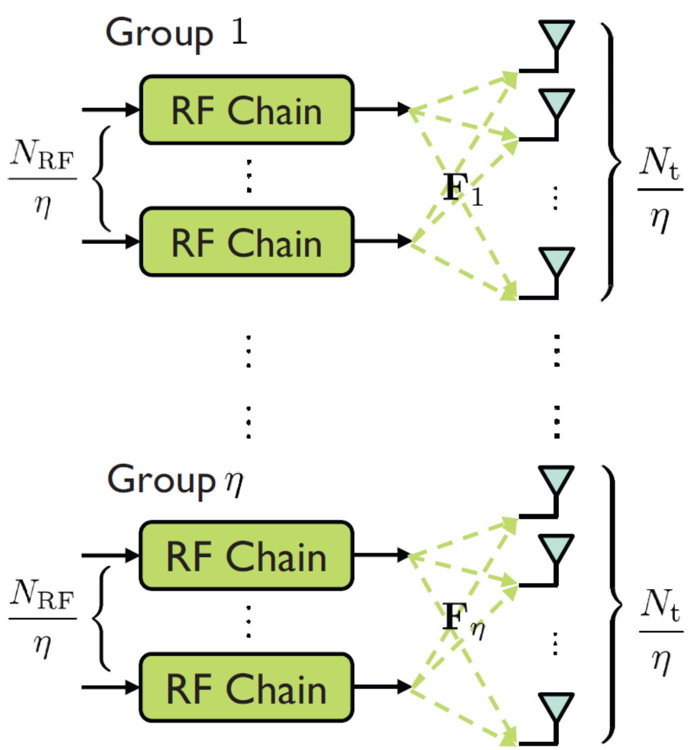
- New mapping strategy
  - Group-connected mapping



$$\mathbf{F}_{\mathrm{RF}} = \begin{bmatrix} \mathbf{R}_1 & \mathbf{0} & \cdots & \mathbf{0} \\ \mathbf{0} & \mathbf{R}_2 & & \mathbf{0} \\ \vdots & & \ddots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & \mathbf{R}_\eta \end{bmatrix}$$

- η =1: fully-connected mapping
- η =N_{RF}: partially-connected

$$\begin{aligned} \underset{\mathbf{F}_i,\mathbf{B}_i}{\text{minimize}} \quad & \|\mathbf{F}_i - \mathbf{R}_i\mathbf{B}_i\|_F^2 \\ \text{subject to} \quad & \mathbf{R}_i \in \mathcal{A}_i, \end{aligned}$$

Designed by directly migrating the design for the fully-connected mapping

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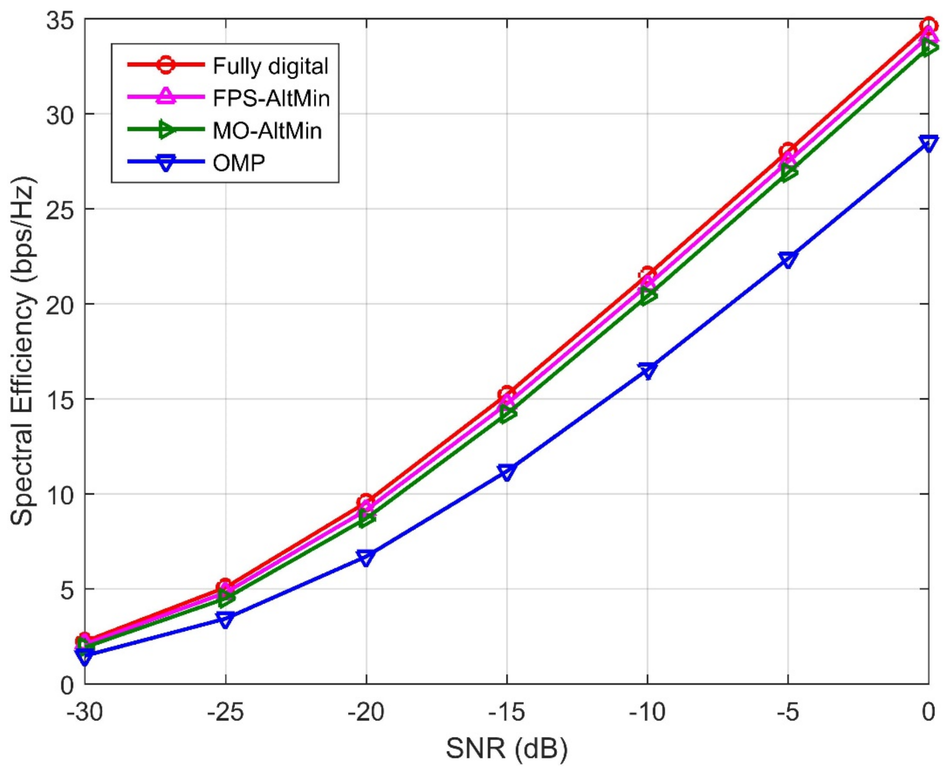
Simulation results: SU-SC systems

$N_{\mathrm{t}} = 144,\ N_{\mathrm{r}} = 16,\ N_{\mathrm{RF}}^{\mathrm{t}} = N_{\mathrm{RF}}^{\mathrm{r}} = N_s = 4,$ and $N_c = 30$



Outperforms the MO-AltMin with lower computational complexity

Effectiveness of the proposed FPS implementation and the FPS-AltMin algorithm

Hybrid Precoding for mm-Wave MIMO Systems

Algorithm Design & Hardware Implementation III: FPS

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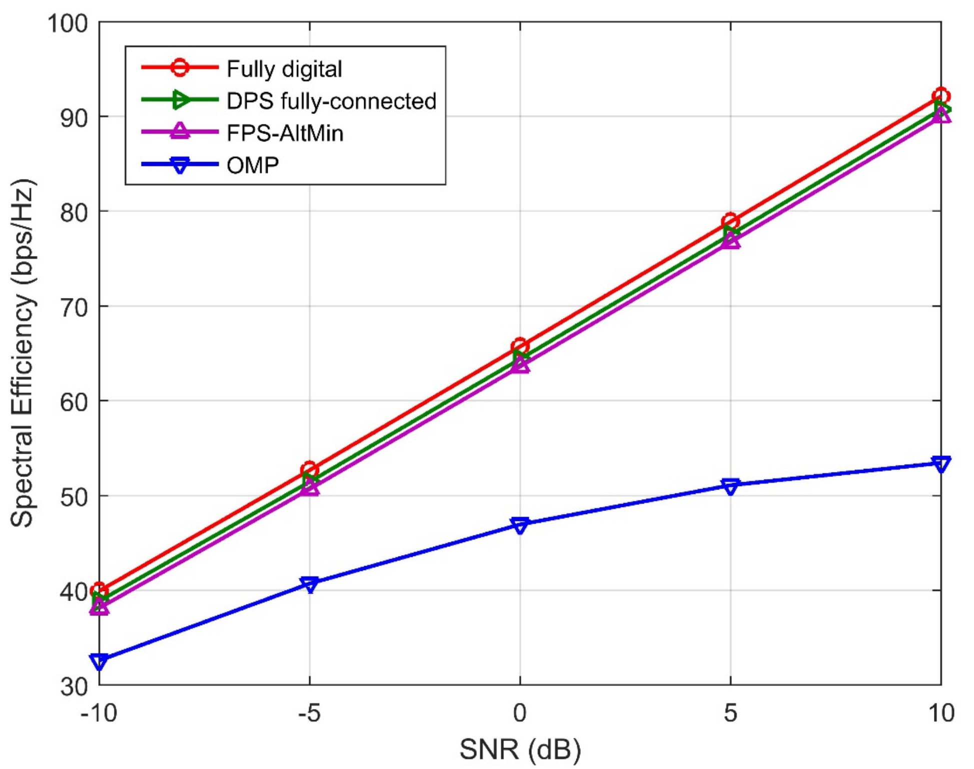
- Simulation results: MU-MC systems

$N_{\mathrm{t}} = 144,\ N_{\mathrm{r}} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{\mathrm{RF}}^{\mathrm{t}} = 8$, and $N_{\mathrm{RF}}^{\mathrm{r}} = 2$



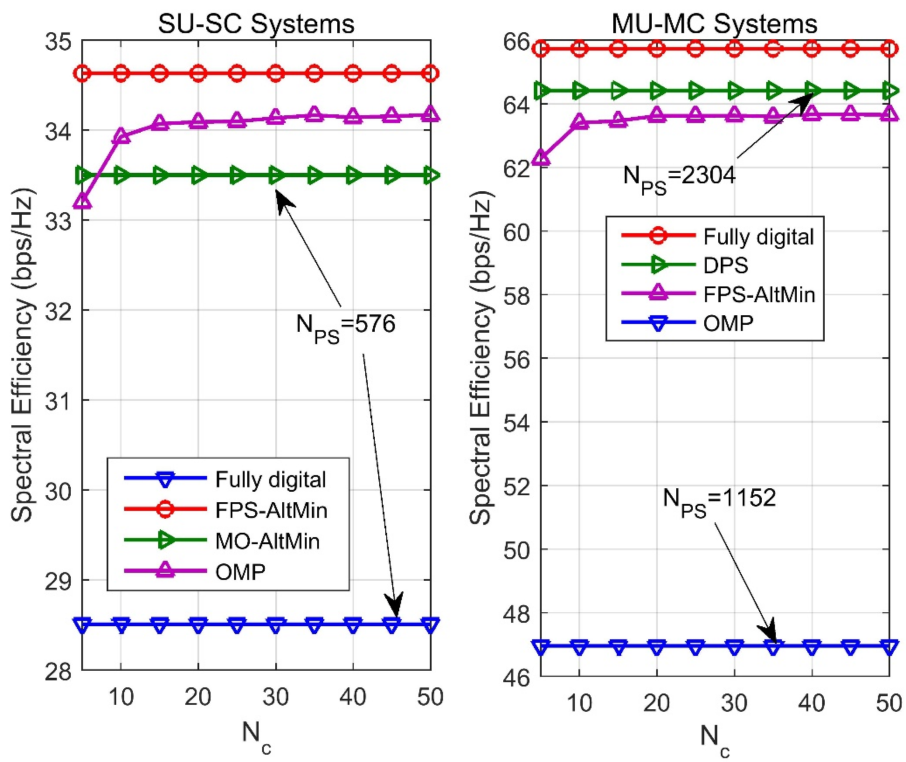
Slightly inferior to the DPS fully-connected mapping with much fewer PSs

Significant improvement over the OMP algorithm

Hybrid Precoding for mm-Wave MIMO Systems
Algorithm Design & Hardware Implementation III: FPS

- Simulation results: How many PSs are needed?

$N_{\mathrm{t}} = 256,\ N_{\mathrm{r}} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{\mathrm{RF}}^{\mathrm{t}} = 8,\ \text{and}\ N_{\mathrm{RF}}^{\mathrm{r}} = 2$



Only ~15 fixed phase shifters are sufficient!

200 times reduction compared with the DPS implementation

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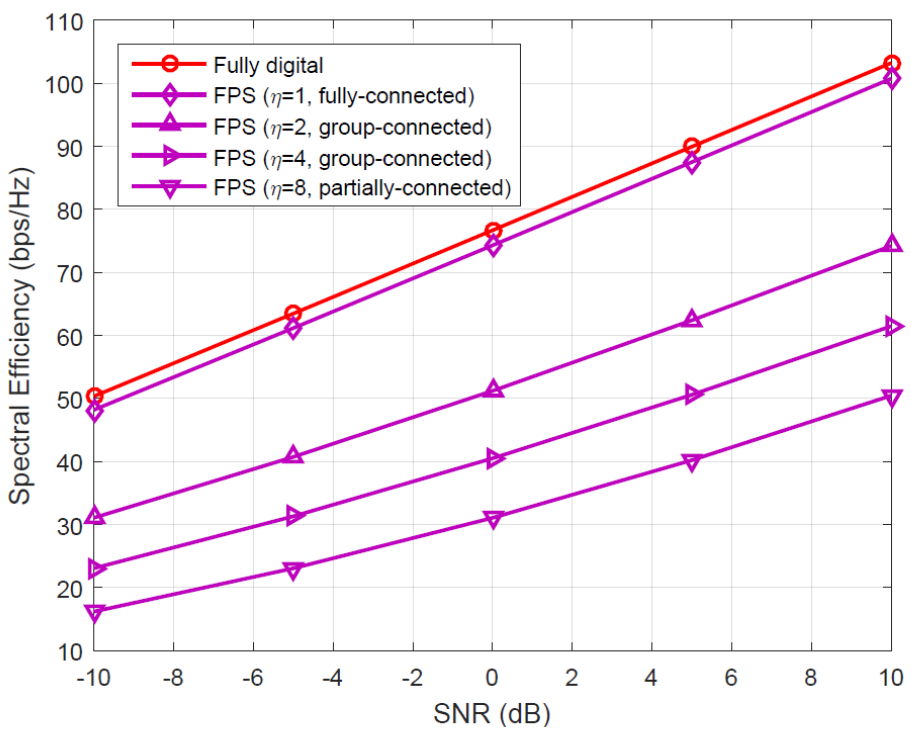
Simulation results

$N_{\mathrm{t}} = 256,\ N_{\mathrm{r}} = 16,\ K = 4,\ F = 128,\ N_s = 2,\ N_{\mathrm{RF}}^{\mathrm{t}} = 8$, and $N_{\mathrm{RF}}^{\mathrm{r}} = 2$



A flexible approach to balance the achievable performance and hardware efficiency

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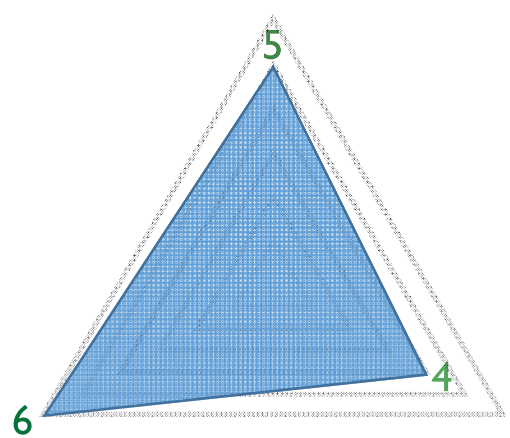
❖ Conclusion



FPS group-connected

Hybrid Precoding for mm-Wave MIMO Systems

Takeaways

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- **Q1**: Can hybrid precoder provide performance close to the fully digital one? **YES**

- **Q2**: How many RF chains are needed? **KN_s**

- **Q3**: How to connect the RF chains and antennas? **Group-connected**

- **Q4**: How many phase shifters are needed? **~15 FPSs**

- **Q5**: How to efficiently design hybrid precoding algorithms?
  Alternating minimization, implementation dependent

Hybrid Precoding for mm-Wave MIMO Systems
Takeaways

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- Comparisons

Imple-mentation	Structure	Design approach	Hardware complexity (No. of phase shifters)	Computational complexity	Performance
SPS	Fully-connected	MO-AltMin	$N_{\mathrm{RF}}^{\mathrm{t}} N_{\mathrm{t}}$	Extremely high	✓✓✓
	Partially-connected	SDR-AltMin	N_{t}	High	✓
DPS	Fully-connected	Matrix decomposition	$2N_{\mathrm{RF}}^{\mathrm{t}}(N_{\mathrm{t}} - N_{\mathrm{RF}}^{\mathrm{t}})$	$\mathcal{O}\left({N_{\mathrm{RF}}^{\mathrm{t}}}^2 N_{\mathrm{t}} F\right)$	✓✓✓✓✓
	Partially-connected	Modified K-means	$2N_{\mathrm{t}}$	$\mathcal{O}\left(N {N_{\mathrm{RF}}^{\mathrm{t}}}^2 N_{\mathrm{t}} F\right)$	✓✓
FPS		FPS-Alt-Min	$N_c \ll N_{\mathrm{t}}$	Medium	✓✓✓✓

- The current **SPS implementation** is not a good choice due to the high design complexity caused by the strict hardware limitation

- The **DPS implementation** is an excellent candidate if high-performance hybrid precoders are required with low design complexity

- The **FPS implementation** finds a trade-off between the hardware and computational complexity, while with satisfactory performance

The Hybrid Precoder Landscape

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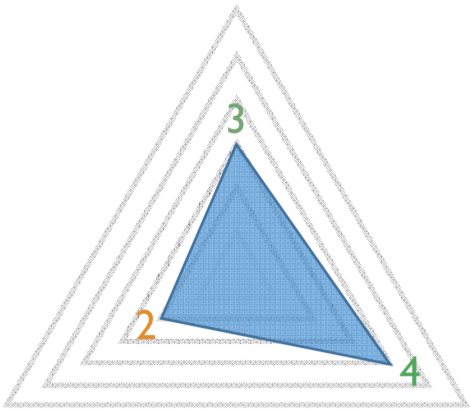
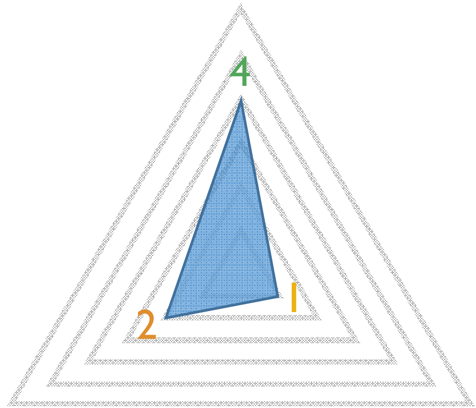
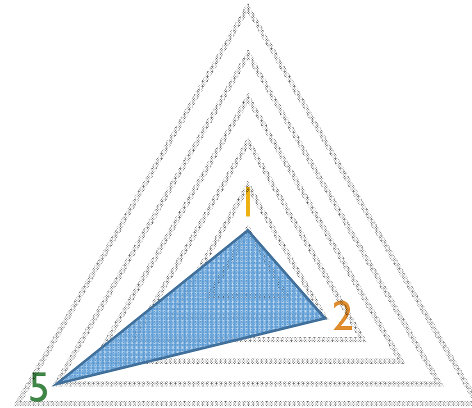


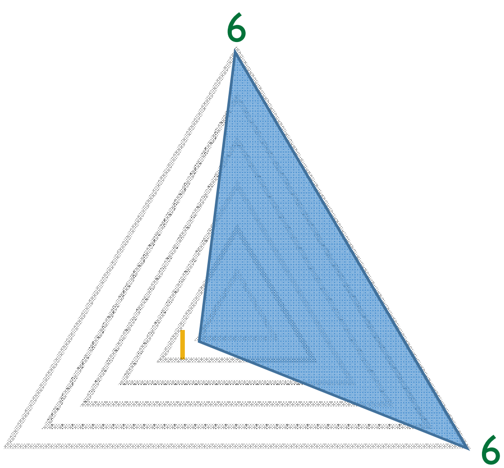
Baseline: SPS fully-connected with OMP



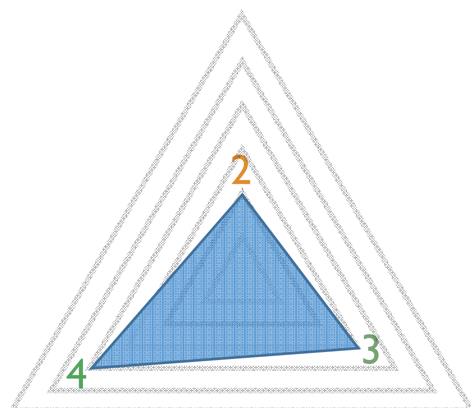
SPS fully-connected



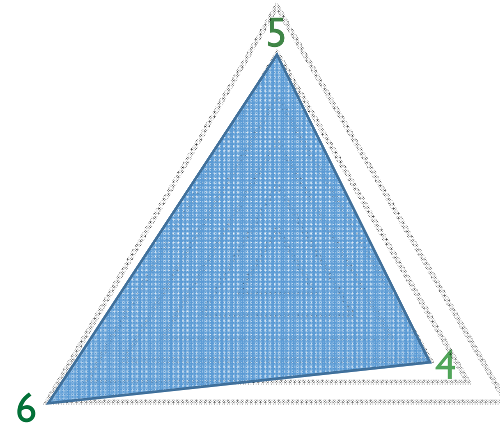
SPS partially-connected



DPS fully-connected



DPS partially-connected



FPS group-connected

Hybrid Precoding for mm-Wave MIMO Systems

Future Research Directions

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- Joint design with CSI acquisition
- Performance evaluation of hybrid precoding algorithms
- Further reduction in computational complexity
- Hardware implementation and testing
- Hybrid precoding with low-precision ADCs
- ……

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Thanks