Tractable Analysis of Large-scale Multi-antenna Wireless Networks via Stochastic Geometry

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Outline

- Background and Motivation
- Fundamentals of Wireless Networks
- A General Tractable Framework
- Analysis of Multi-Antenna Wireless Networks
- Optimization of Multi-Antenna Wireless Networks
- Conclusions

Background and Motivation

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- Era of mobile data deluge

7 x

Data growth by 2021

8.0 Billion

Mobile devices/connections in 2016

60 %

Video traffic in 2016

Cisco VNI, March 2017

Background and Motivation

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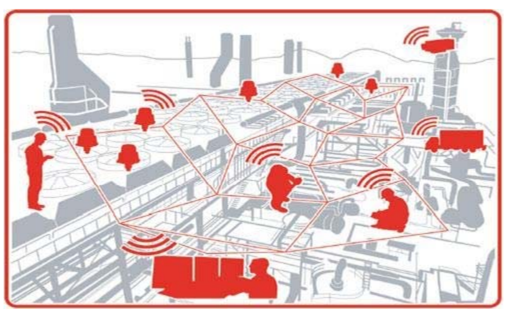
❖ Requirements of 5G systems

High data rate

Massive connections



Uniform coverage

Green communications

Security & privacy

Background and Motivation

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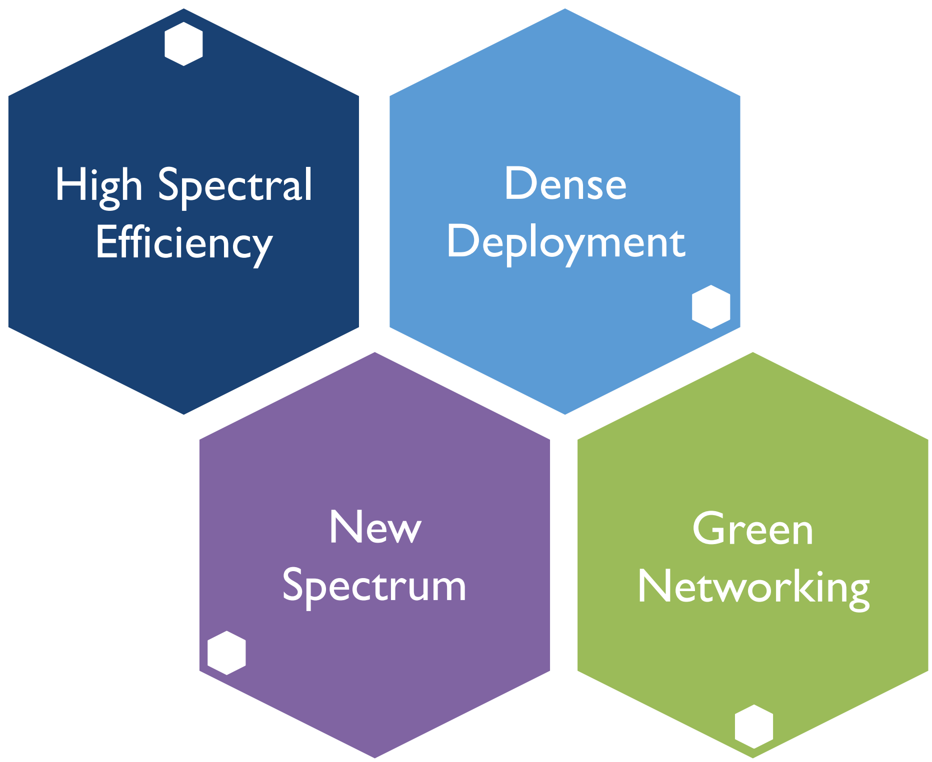
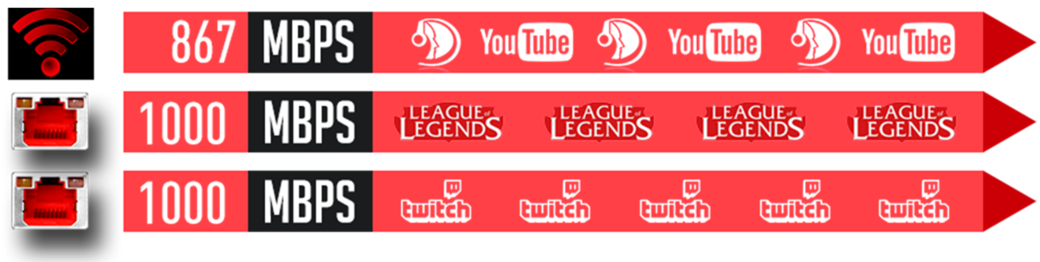
- Approaches to 5G

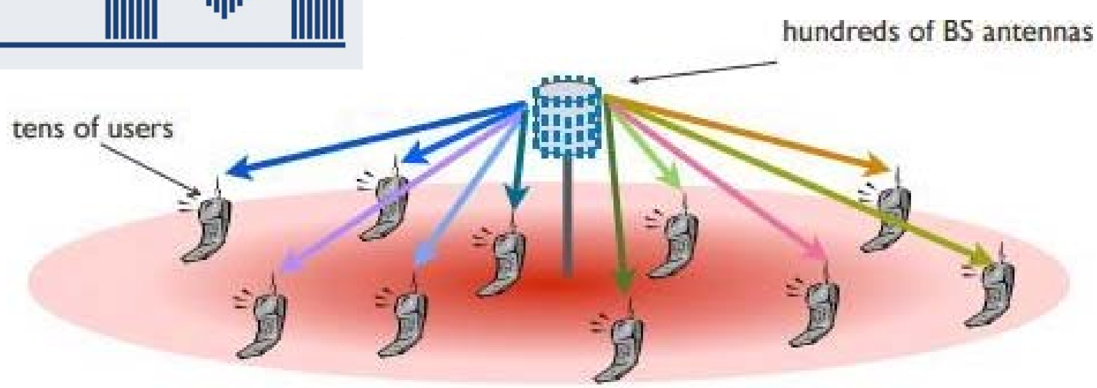


Analytical results for 5G networks to provide design guidance

Background and Motivation

❖ High speed: Spectral efficiency





Background and Motivation

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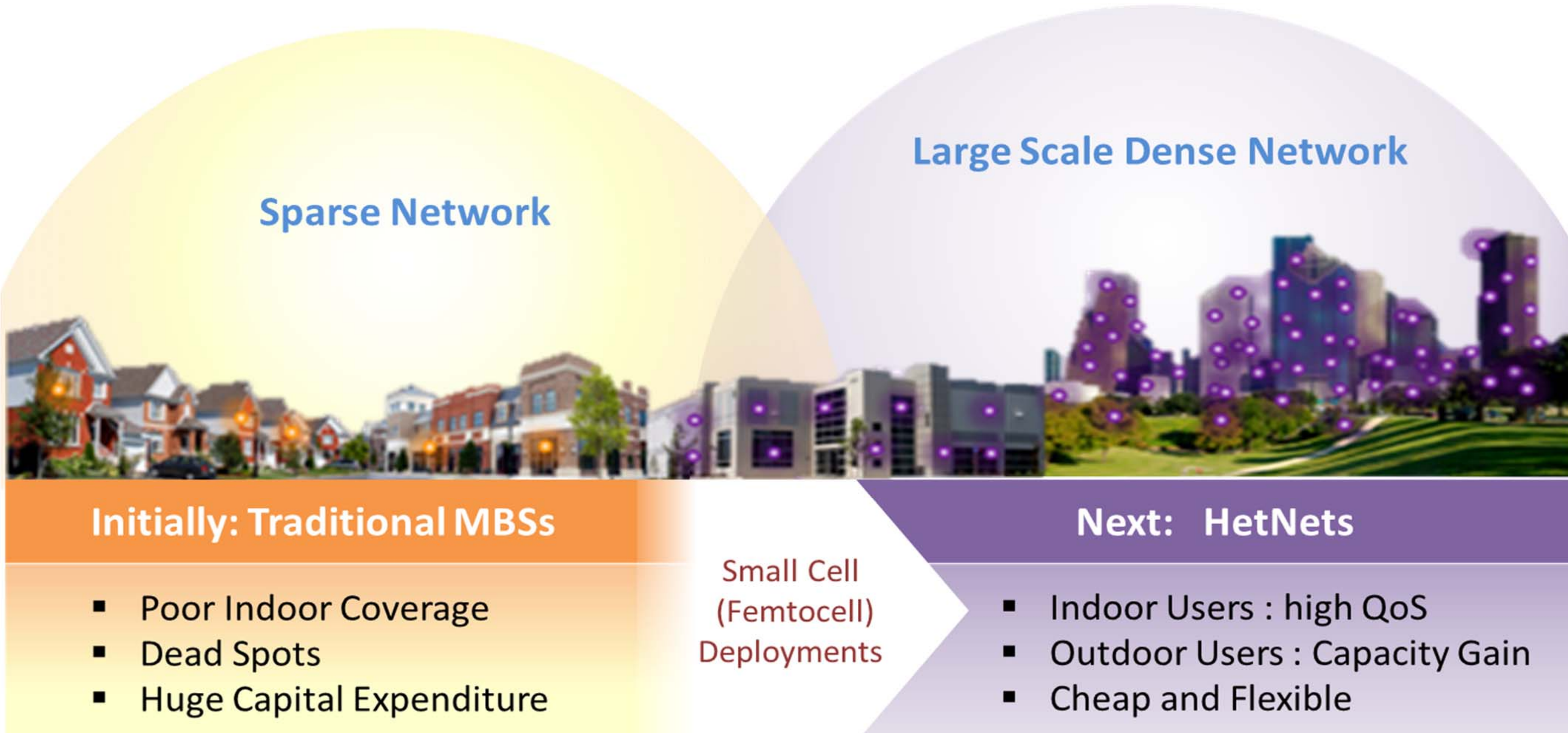
❖ Small cell: Ultra dense networks



Background and Motivation

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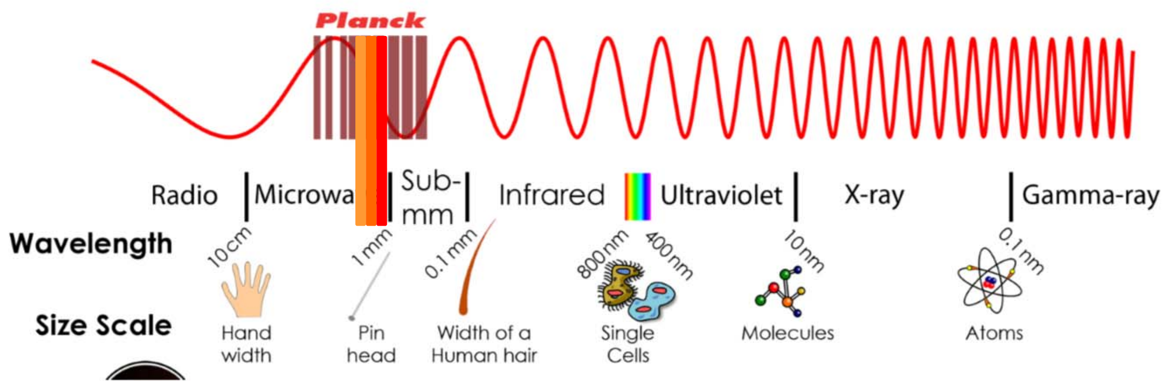
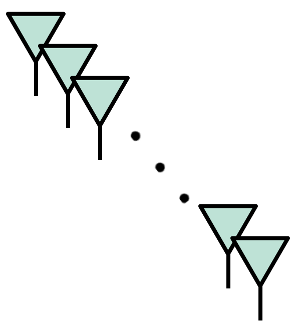
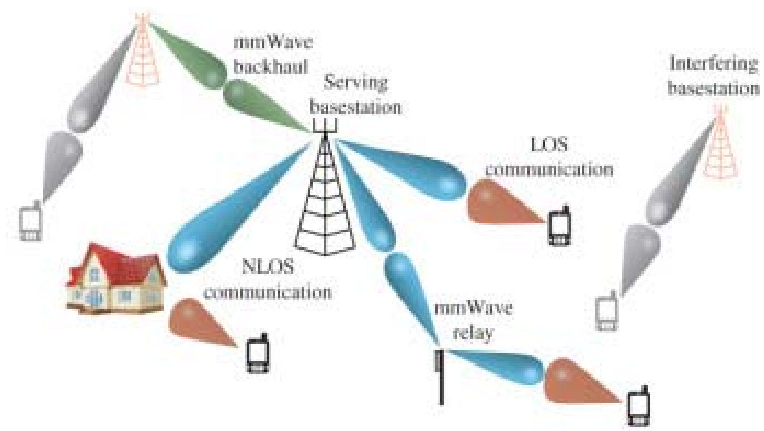
❖ New Spectrum: Beyond Sub-6 GHz





Large-scale antenna array

Highly directional beams



Background and Motivation

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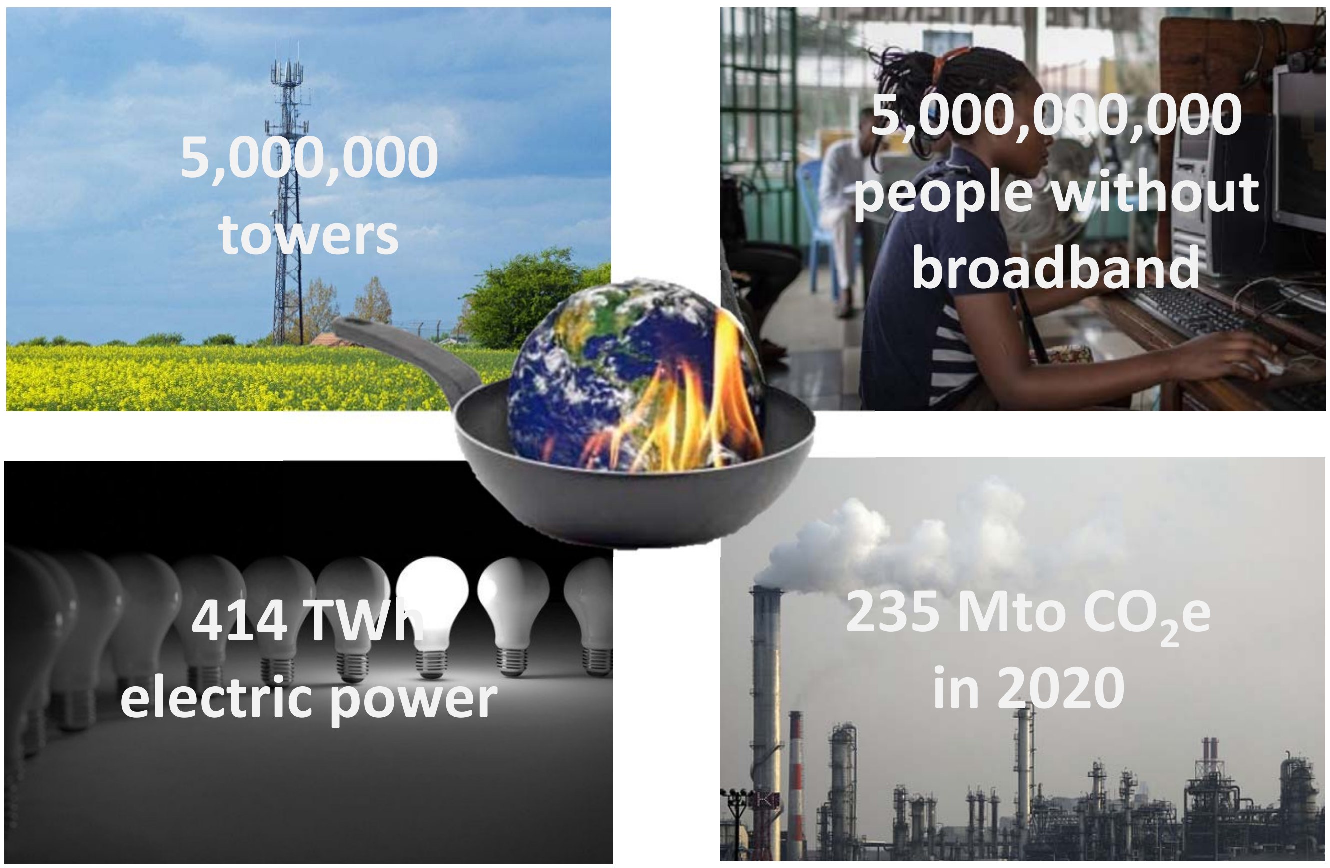
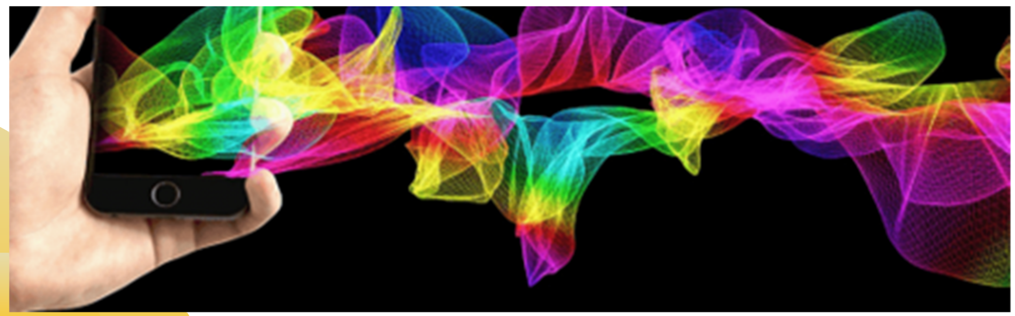
- Green networking: Energy efficiency



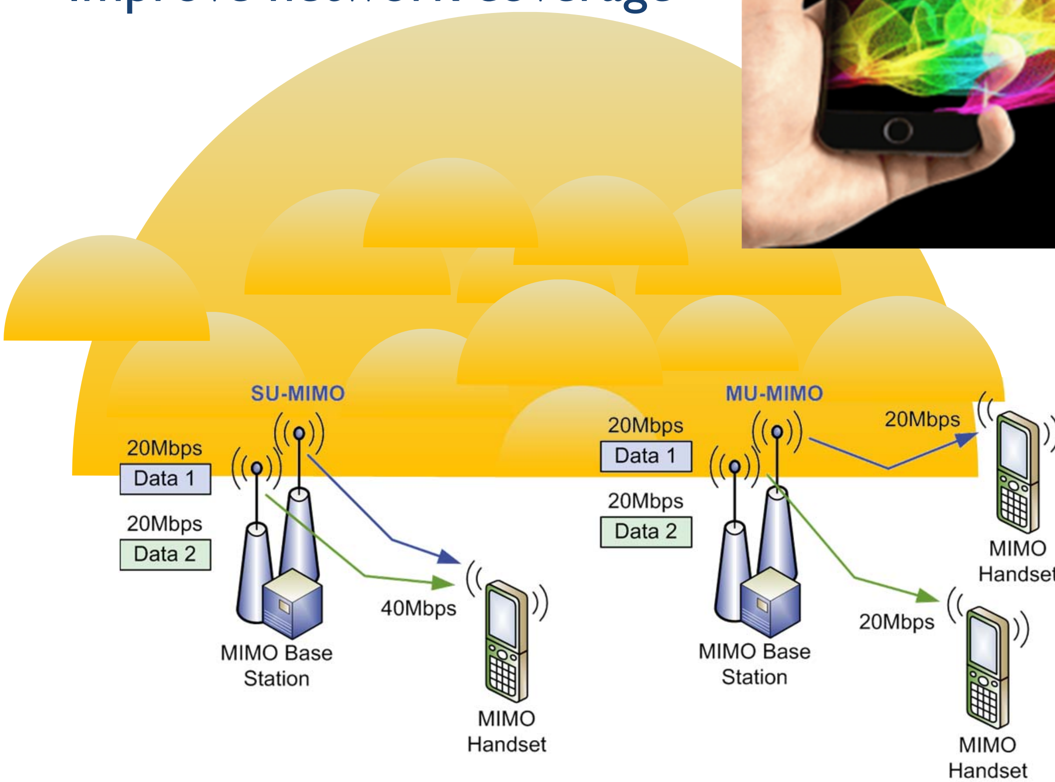
Background and Motivation

❖ The need for multi-antenna (MIMO) techniques

Suppress interference



Improve network coverage



Multiple data streams & SDMA

Background and Motivation

- Key words
  - Dense networks
  - Multi-antenna transmissions

→

Mathematical analysis of multi-antenna wireless networks

- Quick evaluation of different PHY/MAC techniques
- Expose salient network properties
- Avoid building and running system-level simulations

Background and Motivation

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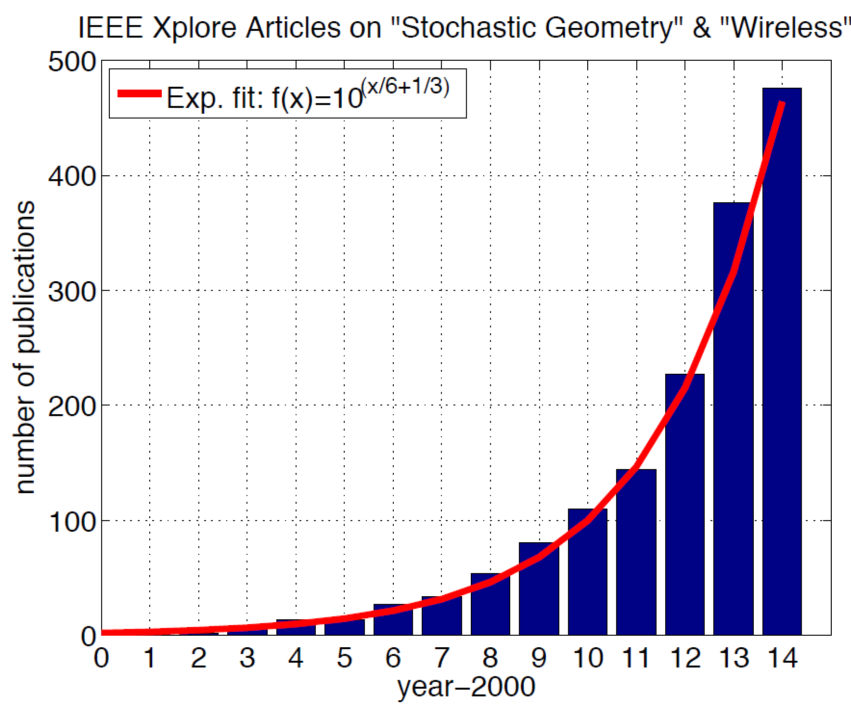
- Tools for evaluating wireless networks

Stochastic geometry

- Applications
  - Cellular networks
  - Ad hoc networks
  - HetNets
  - Cognitive radio
  - D2D
  - Etc.



From Martin Haenggi

Background and Motivation

- Limitations of existing works
  - Closed-form results only available for single-antenna networks with simple transmission schemes and channel models
  - Complicated analytical forms for more general network settings
  - Cannot provided too much network design insights

Need a tractable approach for characterizing multi-antenna 5G networks!

Background and Motivation

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- Key questions to be answered in this tutorial

  - How difficult is it to analyze multi-antenna networks?
  - How will **network densification** affect key performance metrics?
  - How will the **antenna size** affect key performance metrics?
  - What **guidance** can analytical results provide for network design?

Fundamentals of Wireless Networks

Fundamentals of Wireless Networks

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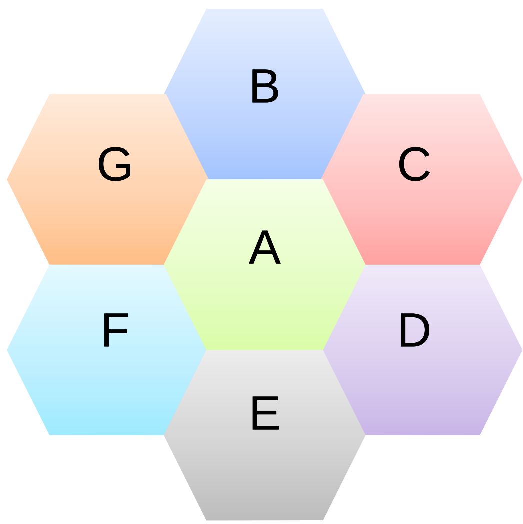
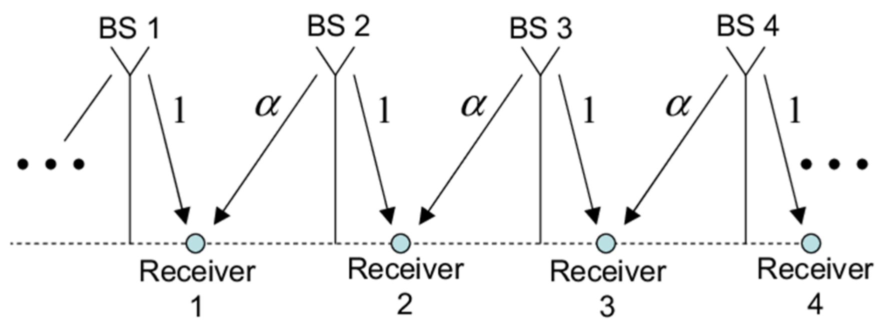
- Previous approaches for network analysis
  - Grid model
  - Wyner model





  - Either less accurate or weakens the analytical tractability

[Ref] J. Xu, **J. Zhang**, and J. G. Andrews, "On the accuracy of the Wyner model in cellular networks," *IEEE Trans. Wireless Commun.*, vol. 10, no. 9, pp. 3098-3109, Sept. 2011.

Fundamentals of Wireless Networks

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❖ How to model large-scale cellular networks?

3122 IEEE TRANSACTIONS ON COMMUNICATIONS, VOL. 59, NO. 11, NOVEMBER 2011

A Tractable Approach to Coverage and Rate in Cellular Networks

Jeffrey G. Andrews, *Senior Member, IEEE*, François Baccelli, and Radha Krishna Ganti, *Member, IEEE*

A tractable approach to coverage and rate in cellular networks

[PDF] from arxiv.org
Find@HKUST Library

Authors	Jeffrey G Andrews, François Baccelli, Radha Krishna Ganti
Publication date	2011/11
Journal	IEEE Transactions on communications
Volume	59
Issue	11
Pages	3122-3134
Publisher	IEEE
Description	Cellular networks are usually modeled by placing the base stations on a grid, with mobile users either randomly scattered or placed deterministically. These models have been used extensively but suffer from being both highly idealized and not very tractable, so complex system-level simulations are used to evaluate coverage/outage probability and rate. More tractable models have long been desirable. We develop new general models for the multi-cell signal-to-interference-plus-noise ratio (SINR) using stochastic geometry. Under very general assumptions, the resulting expressions for the downlink SINR CCDF (equivalent to the coverage probability) involve quickly computable integrals, and in some practical special cases can be simplified to common integrals (eg, the Q-function) or even to simple closed-form expressions. We also derive the mean rate, and then the coverage gain (and mean ...
Total citations	Cited by 1765

Apr. 3, 2018

Fundamentals of Wireless Networks

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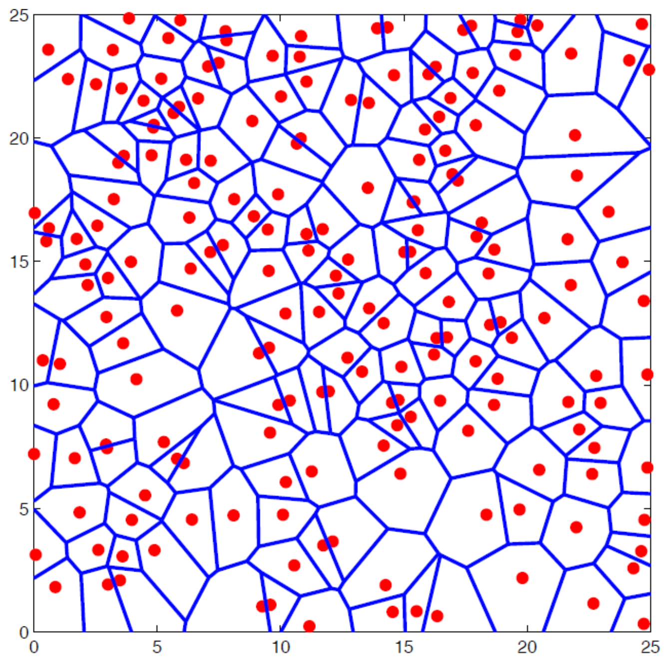
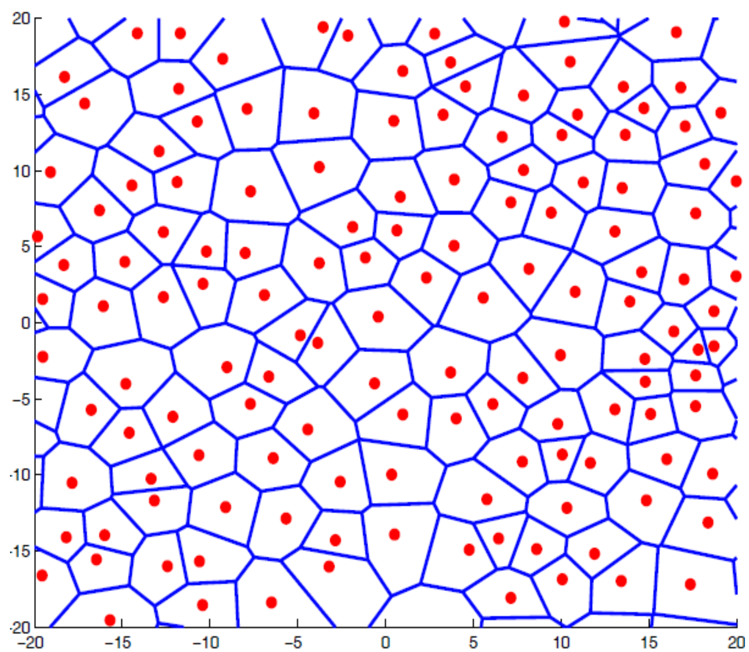
❖ Modeling cellular networks via Poisson point process

Poisson distributed base stations



Actual BS locations in a 4G network



[Ref] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122-3134, Nov. 2011.

Fundamentals of Wireless Networks

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❖ Earlier applications of stochastic geometry

- F. Baccelli and S. Zuyev, "Stochastic geometry models of mobile communication networks," *in Frontiers in Queueing*. CRC Press, 1997, pp. 227–243.
- F. Baccelli, M. Klein, M. Lebourges, and S. Zuyev, "Stochastic geometry and architecture of communication networks," *J. Telecommun. Syst.*, vol. 7, no. 1, pp. 209–227, 1997.
- S. Weber, J. G. Andrews, X. Yang, and G. de Veciana, "Transmission capacity of wireless ad hoc networks with successive interference cancellation," *IEEE Trans. Inf. Theory*, vol. 53, no. 8, pp. 2799–2814, Aug. 2007.
- F. Baccelli, B. Blaszczyszyn, and P. Muhlethaler, "Stochastic analysis of spatial and opportunistic Aloha," *IEEE J. Sel. Areas Commun.*, pp. 1105–1119, Sep. 2009.

Fundamentals of Wireless Networks

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❖ Key references to start

- M. Haenggi, J. G. Andrews, F. Baccelli, O. Dousse, and M. Franceschetti, "Stochastic geometry and random graphs for the analysis and design of wireless networks," *IEEE J. Sel. Areas Commun.*, vol. 27, no. 7, pp. 1029–1046, Sep. 2009.

- F. Baccelli and B. Blaszczyszyn, *Stochastic Geometry and Wireless Networks*. NOW: Foundations and Trends in Networking, 2010.

- M. Haenggi and R. K. Ganti, *Interference in Large Wireless Networks*. NOW: Foundations and Trends in Networking, 2009.

- M. Haenggi, *Stochastic Geometry for Wireless Networks*. Cambridge, U.K.: Cambridge University Press, 2012.

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- Poisson point process (PPP)
  - A stationary PPP of density λ is characterized by the following two properties:

The number of points in any set $B \subset \mathbb{R}^d$ is a Poisson random variable with mean $|B|\lambda$

The number of points in disjoint sets are independent random variables

$$\mathbb{P}(\Phi(B) = k) = e^{-\lambda|B|}\frac{(\lambda|B|)^k}{k!}$$

  - Nice properties for analysis

The superposition of two PPPs of densities λ_1 and λ_2 results in a PPP of density $\lambda_1 + \lambda_2$

Thinning of a PPP: Selecting a point of the process it with probability $1-p$ results in two independent PPPs of intensity measures $p\lambda$ and $(1-p)\lambda$

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- Poisson point process (PPP)
  - Probability generating functional (PGFL)

$$\mathcal{G}[f] = \mathbb{E} \prod_{x \in \Phi} f(x) = \exp\left(- \int_{\mathbb{R}^d} [1 - f(x)] \Lambda(\mathrm{d}x) \right)$$

For stationary point process, the intensity measure $\Lambda(B) = \lambda |B|$

  - A PGFL is very useful to evaluate the Laplace transform

$$\mathbb{E} \exp\left(-s \sum_{x \in \Phi} f(x) \right) = \mathbb{E} \prod_{x \in \Phi} \exp\left(-s f(x) \right)$$
$$= \mathcal{G}[\exp(-s f(\cdot))]$$

    - Laplace transform is critical in performance analysis

- Multi-antenna transmission schemes
  - Beamforming and combining vectors for the i-th receiver

    $\mathbf{f}_i$ $\qquad\qquad$ $\mathbf{w}_i^H$

    - Maximal ratio transmission (MRT)

$$\mathbf{f}_i = \frac{\mathbf{h}_i}{||\mathbf{h}_i||}$$

    - Zero-forcing (ZF)

$$\mathbf{f}_i = \frac{\left(\mathbf{I} - \bar{\mathbf{H}}(\bar{\mathbf{H}}^H\bar{\mathbf{H}})^{-1}\bar{\mathbf{H}}^H\right)\mathbf{h}_i}{\left|\left|\left(\mathbf{I} - \bar{\mathbf{H}}(\bar{\mathbf{H}}^H\bar{\mathbf{H}})^{-1}\bar{\mathbf{H}}^H\right)\mathbf{h}_i\right|\right|}$$

    - Maximal ratio combining

$$\mathbf{w}_i = \mathbf{h}_i$$

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- Signal power gain distribution
  - i.i.d. Rayleigh fading channels
    - Maximal ratio transmission (MRT)

$$\mathbf{f}_i = \frac{\mathbf{h}_i}{||\mathbf{h}_i||} \qquad \mathbf{h}_i^H \mathbf{f}_i \sim \mathrm{Gamma}(N_\mathrm{t}, 1)$$

    - Zero-forcing (ZF)

$$\mathbf{f}_i = \frac{\left(\mathbf{I} - \bar{\mathbf{H}}(\bar{\mathbf{H}}^H \bar{\mathbf{H}})^{-1} \bar{\mathbf{H}}^H\right) \mathbf{h}_i}{\left\| \left(\mathbf{I} - \bar{\mathbf{H}}(\bar{\mathbf{H}}^H \bar{\mathbf{H}})^{-1} \bar{\mathbf{H}}^H\right) \mathbf{h}_i \right\|} \quad \mathbf{h}_i^H \mathbf{f}_i \sim \mathrm{Gamma}(\max(N_t - N_{x_0}, 1), 1)$$

    - Maximal ratio combining

$$\mathbf{w}_i = \mathbf{h}_i \qquad \mathbf{w}_i^H \mathbf{h}_i \sim \mathrm{Gamma}(N_\mathrm{r}, 1)$$

Gamma distribution is commonly encountered in multi-antenna systems

Fundamentals of Wireless Networks

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- Performance metrics
  - Coverage/success probability

$$p_c = \mathbb{P}(\mathrm{SINR} > \tau)$$

  - Area spectral efficiency (ASE)

$$\mathrm{ASE} = \lambda p_c \log_2(1+\tau)$$

  - Energy efficiency

$$\eta = \frac{\mathrm{ASE}}{P}$$

  - Transmission capacity (ad hoc)

$$c = \lambda_c p_c \log_2(1+\tau)$$

Fundamental problem: characterizing the distribution of SI(N)R

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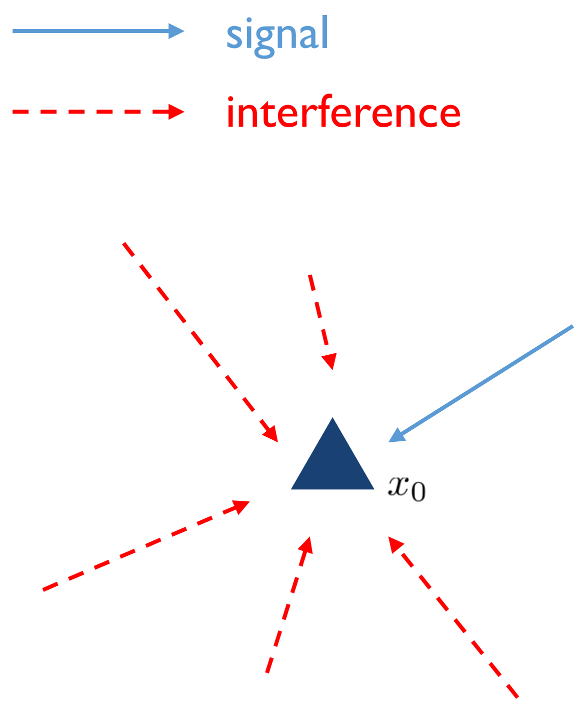
A General Tractable Framework

[Ref] X. Yu, C. Li, **J. Zhang**, K. B. Letaief, "A tractable framework for performance analysis of dense multi-antenna networks," in *Proc. IEEE Int. Conf. Commun. (ICC)*, Paris, France, May 2017.

[Ref] X. Yu, C. Li, **J. Zhang**, M. Haenggi, and K. B. Letaief, "A unified framework for the tractable analysis of multi-antenna wireless networks," submitted.

A General Tractable Framework

❖ Dense network analysis via stochastic geometry



- Transmitters: PPP
- Desired signal:

$$S = P_t g_{x_0} r_0^{-\alpha}$$

- Experienced interference:

$$I = \sum_{x \in \Phi'} P_t g_x ||x||^{-\alpha}$$

A General Tractable Framework

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❖ SINR Expression

$$g_{x_0} = \frac{\left|[\mathbf{W}_{x_0}\mathbf{H}_{x_0}\mathbf{F}_{x_0}]_d\right|^2}{\left[\mathbf{W}_{x_0}\mathbf{W}_{x_0}^H\right]_d} \sim \mathrm{Gamma}(M,\theta)$$

channel power gain
for the desired signal

fixed for ad hoc
random for cellular

distance from the typical receiver to its associated transmitter located at x_0

$$\mathrm{SINR} = \frac{g_{x_0} r_0^{-\alpha}}{\sigma_{\mathrm{n}}^2 + \sum_{x\in\Phi'} g_x \|x\|^{-\alpha}}$$

normalized noise power

$$\left\{(g_x)_{x\in\Phi'_j}\right\}_{j=1}^{J}$$

families of non-negative random variables that are i.i.d. according to arbitrary distributions

$$\Phi' = \cup_{j=1}^{J} \Phi'_j(\lambda_j)$$

J types of interferers

A General Tractable Framework

- Review: Single-antenna networks with Rayleigh fading
  - The signal power gain is exponentially distributed

$$p_c(\tau) = \int_0^\infty f_{r_0}(r)\mathcal{L}(s)\mathrm{d}r$$

$$\mathcal{L}(s) = e^{-s\sigma_n^2}\mathbb{E}_I\left[e^{-sI}\middle| r_0\right] \qquad I \triangleq \sum_{x\in\Phi'} g_x\|x\|^{-\alpha} \qquad s \triangleq \tau r_0^\alpha$$

Proof:

$$p_c(\tau) = \mathbb{P}(\mathrm{SINR} \geq \tau) = \mathbb{P}\left[g_{x_0} \geq s(\sigma_n^2 + I)\right] = \mathbb{E}_{r_0}\left[e^{-s\sigma_n^2}\mathbb{E}_I\left[e^{-sI}|r_0\right]\right]$$

Rayleigh fading

The key is to calculate the Laplace transform!

A General Tractable Framework

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- Review: Single-antenna networks with Rayleigh fading
  - Laplace transform via PGFL

$$\mathbb{E}\exp\left(-s\sum_{x\in\Phi}f(x)\right)=\mathbb{E}\prod_{x\in\Phi}\exp\left(-sf(x)\right)$$
$$=\mathcal{G}[\exp(-sf(\cdot))]$$

Recall $\mathcal{G}[f]=\mathbb{E}\prod_{x\in\Phi}f(x)=\exp\left(-\int_{\mathbb{R}^d}[1-f(x)]\Lambda(\mathrm{d}x)\right)$

$$\mathcal{L}(s)=\exp\left\{-s\sigma_{\mathrm{n}}^2-2\pi\sum_{j=1}^{J}\lambda_j\int_{l_j(r_0)}^{\infty}\left(1-\mathbb{E}_{g_j}[\exp(-sg_jv^{-\alpha})]\right)v\mathrm{d}v\right\}$$

The minimum distance between the typical receiver and the transmitter of the *j*-th type

[Ref] J. G. Andrews, F. Baccelli, and R. K. Ganti, "A tractable approach to coverage and rate in cellular networks," *IEEE Trans. Commun.*, vol. 59, no. 11, pp. 3122-3134, Nov. 2011.

A General Tractable Framework

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- Multi-antenna networks
  - The signal power gain is typically gamma distributed

$$p_c(\tau) = \mathbb{P}\left[g_{x_0} > \tau r_0^\alpha \left(\sigma_n^2 + I\right)\right],$$

where $I \triangleq \sum_{x\in\Phi'} g_x \|x\|^{-\alpha}$.

$$p_c(\tau) = \mathbb{E}_{r_0}\left\{\sum_{n=0}^{M-1} \frac{(\tau r_0^\alpha/\theta)^n}{n!} \mathbb{E}_I\left[\left(\sigma_n^2 + I\right)^n e^{-\frac{\tau r_0^\alpha}{\theta}(\sigma_n^2+I)} \middle| r_0\right]\right\}$$
$$= \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{(-s)^n}{n!} \mathcal{L}^{(n)}(s)\right]$$

  - The key is to calculate the n-th derivative of the Laplace transform! – Highly non-trivial to get tractable expressions!

A General Tractable Framework

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- Existing approaches
  - Bell polynomials

$$p_c(\tau) = \lambda \sum_{n=0}^{M-1} \frac{1}{n!} \sum_{p=0}^{n} B_{n,p}(x_1, \cdots, x_{n-p+1}) \frac{p!(2\tau^\delta)^p}{(\lambda\tau^\delta \mathcal{C} + \lambda)^{p+1}}$$

where

$$x_i = \frac{\delta\lambda}{2} \frac{(\kappa + i - 1)!}{(\kappa - 1)!} B'\left(\kappa + \delta, i - \delta, \frac{1}{1+\tau}\right),$$

$$\mathcal{C} = \delta \sum_{i=1}^{\kappa} \binom{\kappa}{i} B'\left(\kappa - i + \delta, i - \delta, \frac{1}{1+\tau}\right),$$

$B_{n,p}(x_1, \cdots, x_{n-p+1})$ is the incomplete exponential Bell polynomials, and the $B'(a, b, c)$ is the complementary in complete Beta function.

  - Complicated relations between Bell polynomials

[Ref] A. K. Gupta, H. S. Dhillon, S. Vishwanath, and J. G. Andrews, "Downlink multi-antenna heterogeneous cellular network with load balancing," *IEEE Trans. Commun.*, vol. 62, no. 11, pp. 4052–4067, Nov. 2014.

A General Tractable Framework

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- Existing approaches
  - Stirling numbers

$$p_{\mathrm{c}}(\tau)=\frac{(-1)^{M-1}e^{-\lambda r_0^2\left(\frac{\tau\beta}{\theta}\right)^{\delta}\eta(\kappa)-\tau}}{\Gamma(M)}\sum_{l=0}^{M-1}\binom{M-1}{l}r_0^{\frac{2l}{\alpha}}\left(-\frac{\tau}{\theta}\right)^{l}\times \sum_{i=0}^{M-l-1}s(M-l,i+1)\delta^{i}\sum_{j=0}^{i}S(i,j)\left[-\lambda r_0^2\left(\frac{\tau\beta}{\theta}\right)^{\delta}\eta(\kappa)\right]^{j}, \quad (1)$$

where

$$\eta(\kappa)=\frac{\pi\Gamma(\kappa+\delta)\Gamma(1-\delta)}{\Gamma(\kappa)}, \quad (2)$$

while $s(n,k)$ and $S(n,k)$ denote the Stirling numbers of the first and second kind, respectively.

  - Only for ad hoc networks

[Ref] Y. Wu, R. H. Y. Louie, M. R. McKay, and I. B. Collings, "Generalized framework for the analysis of linear MIMO transmission schemes in decentralized wireless ad hoc networks," *IEEE Trans. Wireless Commun.*, vol. 11, no. 8, pp. 2815–2827, Aug. 2012.

A General Tractable Framework

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- Laplace to log-Laplace transform

$$p_{\mathrm{c}}(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{(-s)^n}{n!} \mathcal{L}^{(n)}(s)\right]$$

- Laplace transform

$$\mathcal{L}(s) = \exp\left\{-s\sigma_{\mathrm{n}}^2 - 2\pi \sum_{j=1}^{J} \lambda_j \int_{l_j(r_0)}^{\infty} \left(1 - \mathbb{E}_{g_j}\left[\exp(-s g_j v^{-\alpha})\right]\right) v \mathrm{d}v\right\}$$
$$\triangleq \exp\{\eta(s)\}$$

Its n-th derivative can be calculated via Faà di Bruno’s formula or Bell polynomials, but with unwieldy expressions!

- Log-Laplace transform $\eta(s)$

⇒ tractable results for coverage probability

A General Tractable Framework

- Reveal underlying relations

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{(-s)^n}{n!} \mathcal{L}^{(n)}(s)\right]$$

Lemma 1 Defining $p_n = \frac{(-s)^n}{n!}\mathcal{L}^{(n)}(s)$, there exist recursive relations between $\{p_n\}_{n=0}^{\infty}$, given by

$$p_n = \sum_{i=0}^{n-1} \frac{n-i}{n} t_{n-i} p_i,$$

where

$$t_k = \frac{(-s)^k}{k!}\eta^{(k)}(s).$$

The k-th derivative of log-Laplace transform

> This lemma transforms the calculation of $\mathcal{L}^{(n)}(s)$ to $\eta^{(n)}(s)$

A General Tractable Framework

- Coverage Probability Representation I
  - Finite Sum Representation

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{(-s)^n}{n!} \mathcal{L}^{(n)}(s)\right] \Rightarrow p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} p_n\right] \triangleq \sum_{n=0}^{M-1} \bar{p}_n$$

$$\bar{p}_n = \mathbb{E}_{r_0}[p_n]$$

  - M is typically related to the number of antennas
  - Reveal insights on impacts of multiple antennas
  - Recursive relations $p_n = \sum_{i=0}^{n-1} \frac{n-i}{n} t_{n-i} p_i$
  - Properties mainly rely on p_0, e.g., monotonicity, convexity, etc.

A General Tractable Framework

- Reveal underlying relations

$$p_n = \sum_{i=0}^{n-1} \frac{n-i}{n} t_{n-i} p_i \qquad t_k = \frac{(-s)^k}{k!} \eta^{(k)}(s)$$

  - The recursive relationship is tedious to calculate

    p_n is tedious to calculate, t_k is relatively easy

  - The following lemma leads to an explicit expression for $p_c(\tau)$

Lemma 2 Define two power series $T(z) = \sum_{n=0}^{\infty} t_n z^n$, $P(z) = \sum_{n=0}^{\infty} p_n z^n$. They are related as

$$P(z) = e^{T(z)}.$$

A General Tractable Framework

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❖ Coverage Probability Representation II

➢ l_1-Toeplitz Matrix Representation

Lemma 2

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} p_n\right] = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{1}{n!} P^{(n)}(z)\Big|_{z=0}\right] = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} \frac{1}{n!}\frac{\mathrm{d}^n}{\mathrm{d}z^n} e^{T(z)}\Big|_{z=0}\right]$$

Fact I: the n-th term is determined by the n-th coefficient of $e^{T(z)}$

Fact II: the first M coefficients of $e^{T(z)}$ form the first column of $e^{\mathbf{T}_M}$ [Henrici 1988]

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\left\|e^{\mathbf{T}_M}\right\|_1\right] \qquad \mathbf{T}_M = \begin{bmatrix} t_0 & & & & \\ t_1 & t_0 & & & \\ t_2 & t_1 & t_0 & & \\ \vdots & & & \ddots & \\ t_{M-1} & \cdots & t_2 & t_1 & t_0 \end{bmatrix}$$

$\|\mathbf{A}\|_1 = \max_{1\le j\le n} \sum_{i=1}^{m} |a_{ij}|$

➢ A compact expression for numerical evaluation

➢ *Matrix representation better suits MIMO*

A General Tractable Framework

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❖ Coverage Probability Representation II

➢ Properties of l_1-Toeplitz matrices help further analysis

i) Matrix exponenital $\exp(\mathbf{T}_M)$ and inverse $\mathbf{T}_M^{-1}$ are also lower triangular Toeplitz matrices

ii) The n-th power of the strictly lower triangular matrix $(\mathbf{T}_M - t_0\mathbf{I}_M)^n = \mathbf{0}$ for $n \geq M$

iii) The partial derivative

$$\frac{\partial ||\mathbf{T}_M||_1}{\partial x} \sim \left\| \frac{\partial \mathbf{T}_M}{\partial x} \right\|_1$$

iv) Norm inequality

$$||\mathbf{T}_M \mathbf{T}'_M||_1 \leq ||\mathbf{T}_M||_1 ||\mathbf{T}'_M||_1$$

A General Tractable Framework

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- Beyond Gamma distribution
  - A general pdf for signal power gain

$$f_{g_{x_0}}(u) = \sum_{p\in\mathcal{P}} e^{-\phi_p u} \sum_{q\in\mathcal{Q}} \varphi_{p,q} u^q$$

$$\mathcal{P}, \mathcal{Q} \subset \mathbb{N}_0,\ \phi_p, \varphi_{p,q} \in \mathbb{R}$$

$$p_c(\tau) = 1 - \sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{Q}} \frac{\varphi_{p,q} q!}{\phi_p^{q+1}} + \sum_{p\in\mathcal{P}}\sum_{q\in\mathcal{Q}} \frac{\varphi_{p,q} q!}{\phi_p^{q+1}} \mathbb{E}_{r_0}\left[\left\| e^{\mathbf{T}_{q+1}^{(p)}} \right\|_1\right]$$

$$t_{p,k} = \frac{(-s_p)^k}{k!}\eta^{(k)}(s_p), \quad 0 \le k \le q$$

$$s_p = \tau r_0^\alpha \phi_p$$

  - Gamma distribution: a special case

$$\mathcal{P} = \{0\},\ \mathcal{Q} = \{M-1\},\ \phi_0 = \tfrac{1}{\theta},\ \text{and}\ \varphi_{0,M-1} = \tfrac{1}{\theta^M \Gamma(M)}$$

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- Steps to apply the proposed framework
  - Calculate the log-Laplace transform according to f_{g_x}

$$\eta(s) = -s\sigma_{\mathrm{n}}^2 - 2\pi \sum_{j=1}^{J} \lambda_j \int_{l_j(r_0)}^{\infty} \left(1 - \mathbb{E}_{g_j}\left[\exp(-s g_j v^{-\alpha})\right]\right) v \mathrm{d}v$$

  - Calculate the derivatives of the log-Laplace transform

$$t_k = \frac{(-s)^k}{k!} \eta^{(k)}(s)$$

    - The only additional step for multi-antenna networks
    - Usually in closed-forms
    - Help reveal insights

The analysis is almost as tractable as single-antenna networks!

A General Tractable Framework

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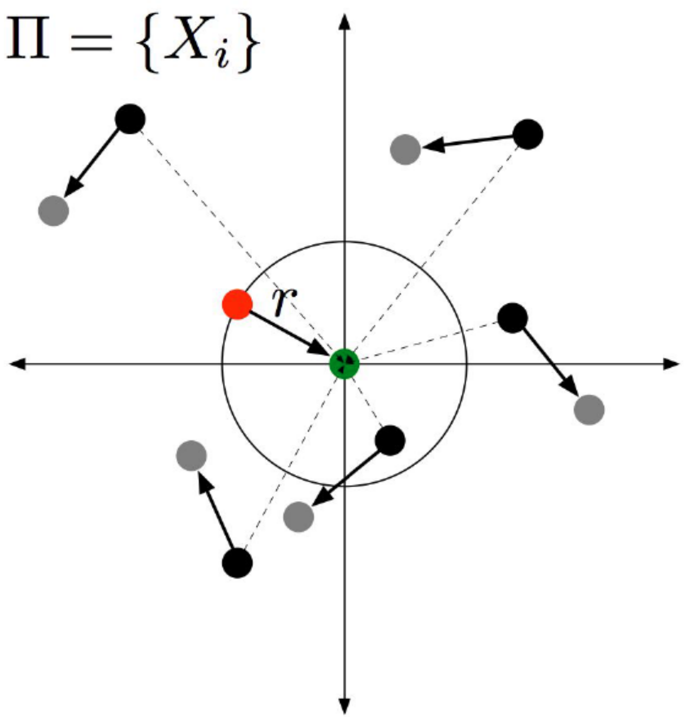
- Ad hoc networks
  - Dipole model



A General Tractable Framework

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- Ad hoc networks

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\left\|e^{\mathbf{T}_M}\right\|_1\right]$$

  - Dipole model
  - Step 1: Calculate the log-Laplace transform

$$\eta(s) = -s\sigma_{\mathrm{n}}^2 - \pi\lambda\Gamma(1-\delta)s^{\delta}\mathbb{E}_g\left[g^{\delta}\right]$$

  - Step 2: Calculate the derivatives of the log-Laplace transform

$$a_n = \frac{(-s)^n}{n!}\eta^{(n)}(s) = \frac{(-1)^n}{n!}\left\{-\mathbb{1}(n\le 1)\frac{\tau r_0^{\alpha}}{\theta}\sigma_{\mathrm{n}}^2 - \pi\lambda r_0^2\Gamma(1-\delta)(\delta)_n\left(\frac{\tau}{\theta}\right)^{\delta}\mathbb{E}_g\left[g^{\delta}\right]\right\}$$

falling factorial

$$p_c(\tau) = \left\|e^{\mathbf{A}_M}\right\|_1 \qquad \delta = \frac{2}{\alpha}$$

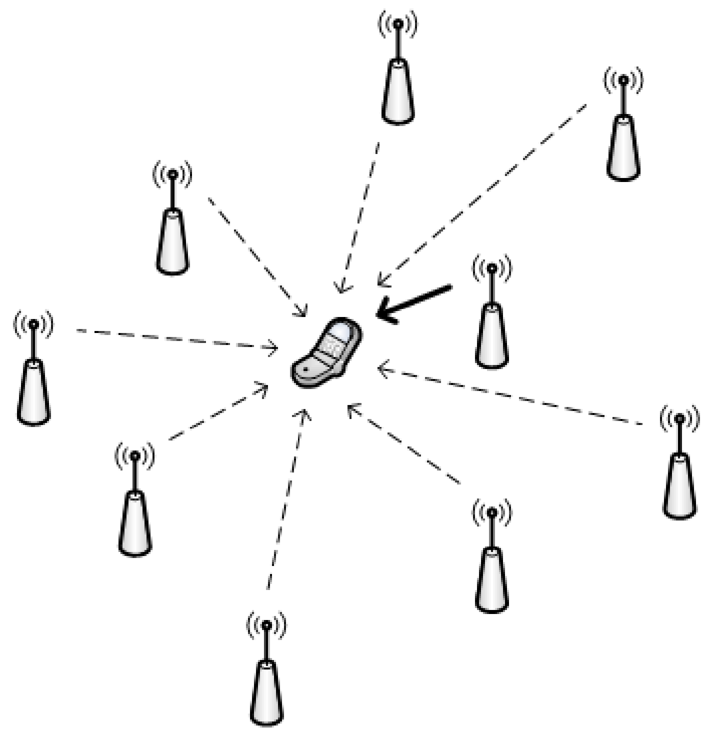
Toeplitz matrix with a_n as elements

Fast algorithm to matrix exponential of lower triangular Toeplitz matrices

- D. Kressner and R. Luce, "Fast computation of the matrix exponential for a Toeplitz matrix," arXiv preprint arXiv: 1607.01733, 2016.

A General Tractable Framework

- Cellular networks
  - The nearest-BS association
  - Consider SIR coverage (interference-limited)



A General Tractable Framework

- Cellular networks

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\left\|e^{\mathbf{T}_M}\right\|_1\right]$$

  - Single-tier

$$f_{r_0}(r) = 2\pi\lambda r e^{-\pi\lambda r^2}$$

The set of interfering BSs $\Phi' = \Phi\backslash\{x_0\}$ forms a PPP on $\mathbb{R}^2\backslash b(0, r_0)$ conditioned on $x_0 \in \Phi$.

  - Step 1: Calculate the log-Laplace transform

$$\eta(s) = -2\pi\lambda\int_{r_0}^{\infty}\left(1 - \mathbb{E}_g[\exp(-sgv^{-\alpha})]\right)v\mathrm{d}v = \pi\lambda r_0^2 - \pi\lambda r_0^2\mathbb{E}_g\left[{}_1F_1\left(-\delta; 1-\delta; -sr_0^{-\alpha}g\right)\right]$$

  - Step 2: Calculate the derivatives of the log-Laplace transform

$$t_n = -\pi\lambda r_0^2\frac{\delta}{\delta - n}\frac{(\tau/\theta)^n}{n!}\left\{\mathbb{E}_g\left[g^n{}_1F_1\left(n-\delta; n+1-\delta; -\frac{\tau}{\theta}g\right)\right] - \mathbb{1}(n=0)\right\}$$

$$p_c(\tau) = \int_0^{\infty} 2\pi\lambda r e^{-\pi\lambda r^2}\left\|e^{\mathbf{T}_M}\right\|_1 \mathrm{d}r$$

A General Tractable Framework

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- Cellular networks $p_c(\tau) = \int_0^\infty 2\pi\lambda r e^{-\pi\lambda r^2} \left\| e^{\mathbf{T}_M} \right\|_1 \mathrm{d}r$
  - Further simplification

Lemma 3 Denote $c_n = \frac{\delta}{\delta-n}\frac{(\tau/\theta)^n}{n!}\mathbb{E}_g\left[g^n {}_1F_1\left(n-\delta; n+1-\delta; -\frac{\tau}{\theta}g\right)\right], \quad 0 \leq n \leq M-1$. Define power serie $C(z) = \sum_{n=0}^\infty c_n z^n$, and then it is related with $\bar{P}(z) = \sum_{n=0}^\infty \bar{p}_n z^n$ as

$$\bar{P}(z) = \frac{1}{C(z)}.$$

  - Following similar argument as slide 39

$$p_c(\tau) = \left\| \mathbf{C}_M^{-1} \right\|_1$$

Toeplitz matrix with c_n as elements

Fast algorithm to calculate the inversion of Toeplitz matrices

- D. Commenges and M. Monsion, "Fast inversion of triangular Toeplitz matrices," *IEEE Trans. Autom. Control*, vol. 29, no. 3, pp. 250–251, Mar. 1984.

A General Tractable Framework

- Unique properties: Effects of densification
  - Ad hoc networks

$$a_n = -\frac{(-1)^n}{n!}(\delta)_n \pi\lambda\Gamma(1-\delta)s^\delta \mathbb{E}_g\left[g^\delta\right] + s\sigma_{\mathrm{n}}^2 \mathbb{1}(n=1)$$

Positive for $n \geq 1$

$$p_n = \sum_{i=0}^{n-1} \frac{n-i}{n} a_{n-i} p_i$$

  - Monotonicity and convexity depend on p_0

$$p_0 = e^{\eta(s)} = \exp\left(-s\sigma_{\mathrm{n}}^2 - \pi\lambda\Gamma(1-\delta)s^\delta \mathbb{E}_g\left[g^\delta\right]\right)$$

which is monotonically decreasing and convex in λ

A General Tractable Framework

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- Unique properties: Effects of densification
  - Ad hoc networks

$$(\mathbf{A}'_M - a'_0\mathbf{I}_M)^n = \mathbf{0} \text{ for } n \geq M$$

$$e^{\mathbf{A}_M} = e^{\lambda \mathbf{A}'_M} = e^{a'_0\lambda} \cdot \sum_{n=0}^{M-1} \frac{1}{n!} \left[\lambda \left(\mathbf{A}'_M - a'_0\mathbf{I}_M\right)\right]^n$$

$$\mathbf{A}'_M = a'_0\mathbf{I}_M + (\mathbf{A}'_M - a'_0\mathbf{I}_M)$$

Explicitly reveal the impact of density

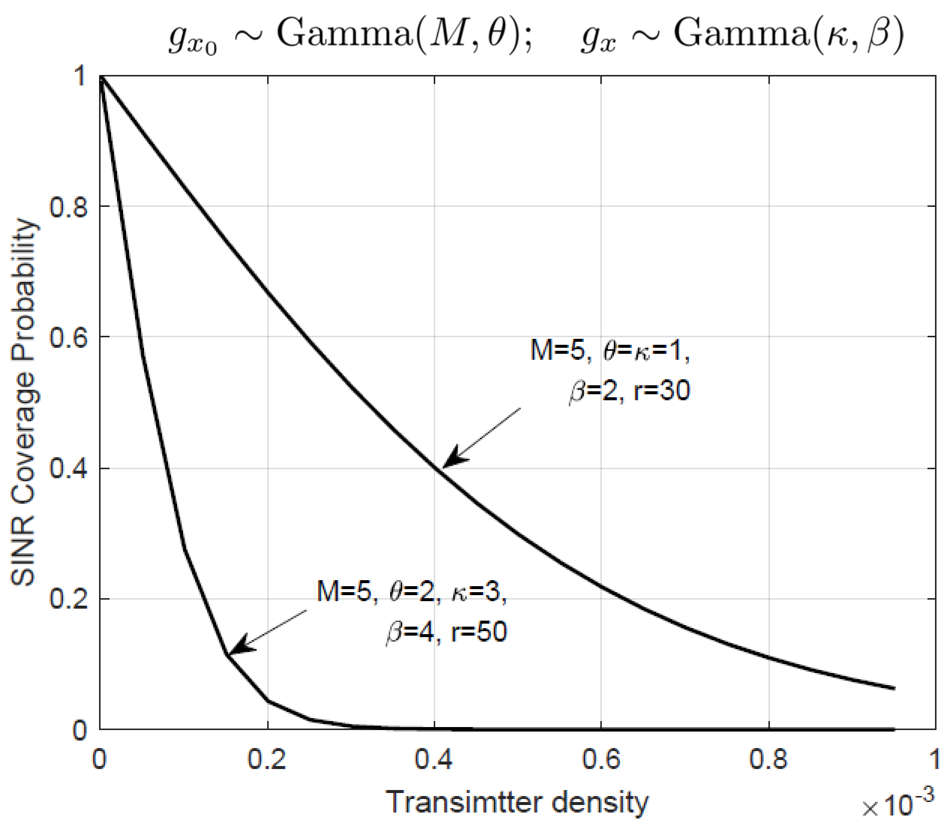
$$p_c(\lambda) = e^{a'_0\lambda} \sum_{n=0}^{M-1} \beta_n \lambda^n$$

$$\beta_n = \frac{\left\|(\mathbf{A}'_M - a'_0\mathbf{I}_M)^n\right\|_1}{n!} \qquad a'_n = \frac{a_n}{\lambda} = -\frac{(-1)^n}{n!}(\delta)_n \pi r_0^2 \Gamma(1-\delta)\left(\frac{\tau}{\theta}\right)^\delta \mathbb{E}_g\left[g^\delta\right]$$

  - A product of an exponential function and a polynomial function of order *M-1* of the transmitter density λ

A General Tractable Framework

- Unique properties: Effects of densification



$$p_c(\lambda) \to 1 \text{ if } \lambda \to 0$$

$$p_c(\lambda) = 1 - \epsilon \to \lambda^*$$

The maximal density for a given coverage requirement

A General Tractable Framework

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- Unique properties: Effects of densification
  - Cellular networks

$$p_c(\tau) = \left\|\mathbf{C}_M^{-1}\right\|_1$$

$$c_n = \frac{\delta}{\delta - n}\frac{(\tau/\theta)^n}{n!}\mathbb{E}_g\left[g^n {}_1F_1\left(n-\delta; n+1-\delta; -\frac{\tau}{\theta}g\right)\right], \quad 0 \le n \le M-1$$

λ does not appear in $p_c(\tau)$

This is previously known for single-antenna networks

SIR invariance holds for single-tier multi-antenna networks

- Cannot be analytically shown via previous complicated results

$$p_c(\tau) = \lambda \sum_{n=0}^{M-1} \frac{1}{n!} \sum_{p=0}^{n} B_{n,p}(x_1(\lambda), \cdots, x_{n-p+1}(\lambda)) \frac{p!(2\tau^\delta)^p}{(\lambda\tau^\delta \mathcal{C} + \lambda)^{p+1}}$$

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❖ Unique properties: Effects of multiple antennas

	Multi-antenna transmission technique ($\mathbf{F}_{x_0}$/$\mathbf{W}_{x_0}$)	**Channel fading ($\mathbf{H}_{x_0}$)**	**Signal power gain (g_{x_0}) distribution**
Throughput and Energy Efficiency Analysis [24]	MRT	Rayleigh	$\mathrm{Gamma}(N_\mathrm{t}, 1)$
Interference Coordination [25]	Partial ZF beamforming	Rayleigh	$\mathrm{Gamma}(\max(N_\mathrm{t} - N_{x_0}, 1), 1)$
SIMO Ad Hoc Networks [14]	Partial ZF combining	Rayleigh	$\mathrm{Gamma}(N_\mathrm{r} - N_{x_0}, 1)$
Spatial Multiplexing in Ad Hoc Networks [22]	Maximum ratio combining (MRC)	Rayleigh	$\mathrm{Gamma}(N_\mathrm{r}, 1)$
Multi-tier Multiuser MIMO HetNets [26]	SDMA	Rayleigh	$\mathrm{Gamma}(N_\mathrm{t} - U + 1, 1)^*$
Physical Layer Security Aware Networks [1]	Jamming & ZF beamforming	Rayleigh	$\mathrm{Gamma}(D, 1)$

➢ The antenna size is typically reflected in the shape parameter *M* of the gamma distribution

A General Tractable Framework

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- Unique properties: Effects of multiple antennas
  - l_1-Toeplitz matrix representation

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\left\|e^{\mathbf{T}_M}\right\|_1\right]$$

Dimension of the matrix

  - Finite sum representation

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} p_n\right]$$

Number of terms in the sum

A General Tractable Framework

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- Unique properties: Effects of multiple antennas
  - Increase the number of antennas

$$p_c(\tau) = \mathbb{E}_{r_0}\left[\sum_{n=0}^{M-1} p_n\right] = \sum_{n=0}^{M-1} \bar{p}_n$$

$$\bar{p}_n = \mathbb{E}_{r_0}\left[e^{t_0}\frac{\|(\mathbf{T}_M - t_0\mathbf{I}_M)^n\|_1}{n!}\right]$$

All the entries in the strict lower triangular matrix $\mathbf{T}_M - t_0\mathbf{I}_M$ are non-negative.

Proposition 1 For both ad hoc and cellular networks, increasing the antenna size always improves the coverage probability, i.e., $\bar{p}_n > 0$ for $n > 0$.

  - The coverage improvement due to the M+1-th antenna is

$$p_c(M+1) - p_c(M) = \bar{p}_M$$

A General Tractable Framework

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- Unique properties: Effects of multiple antennas
  - Cellular Networks

$$p_{\mathrm{o}}(\tau) = 1 - \sum_{n=0}^{M-1} \bar{p}_n$$

Proposition 2 Denoting the outage probability in multi-antenna cellular networks by $p_{\mathrm{o}}(M)$, we have

$$\lim_{M\to\infty} \frac{p_{\mathrm{o}}(M)}{p_{\mathrm{o}}(M+1)} = \lim_{n\to\infty} \frac{\bar{p}_n}{\bar{p}_{n+1}} = r_{\mathrm{c}} > 1,$$

where r_{c} is the radius of convergence of the power series $\bar{P}(z)$, given by the solution to the equation

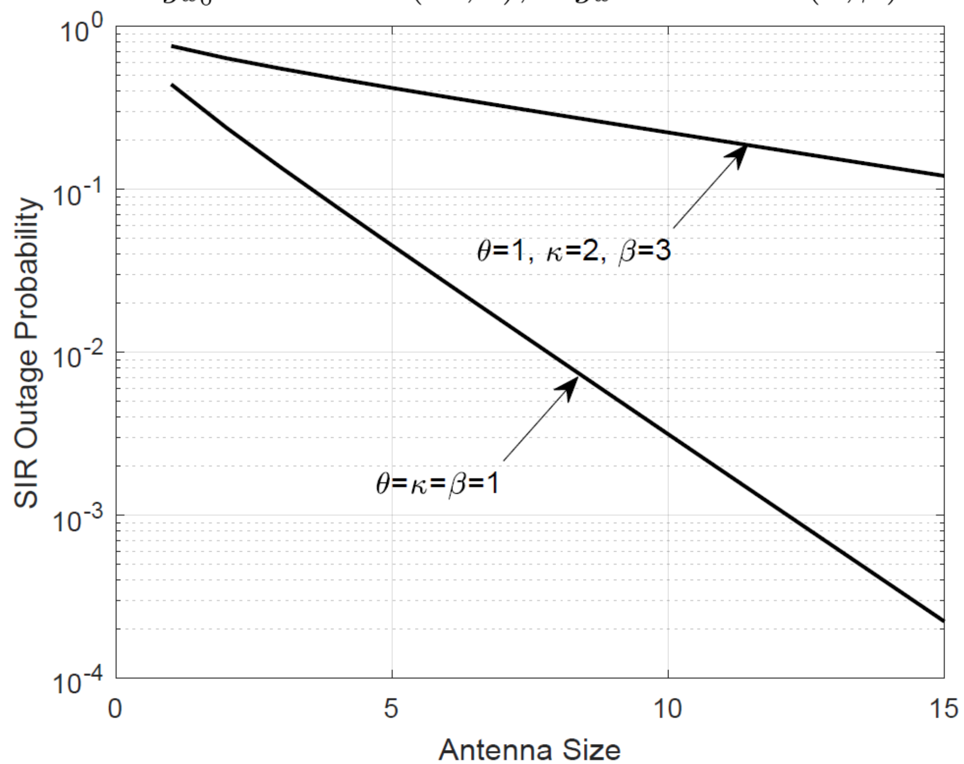
$$\mathbb{E}_g\left[{}_1F_1\left(-\delta; 1-\delta; \frac{(r_{\mathrm{c}}-1)\tau}{\theta} g\right)\right] = 0.$$

Outage probability of cellular networks in the logarithmic scale decrease linearly in M with slope $-\log_{10} r_c$

A General Tractable Framework

- Unique properties: Effects of multiple antennas
  - Cellular Networks

$$g_{x_0} \sim \mathrm{Gamma}(M, \theta); \quad g_x \sim \mathrm{Gamma}(\kappa, \beta)$$



A General Tractable Framework

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- Unique properties: Effects of multiple antennas
  - Ad hoc Networks
  - Special case: α=4

Proposition 3 When the path loss exponent $\alpha = 4$, the SIR coverage improvement due to adding the $n+1$-th antenna in ad hoc multi-antenna networks monotonically decreases in the interval

$$n > \frac{\mu^2}{4} - 1,$$

where μ is given by

$$\mu = \pi\lambda r_0^2\Gamma(1-\delta)\left(\frac{\tau}{\theta}\right)^{\delta}\mathbb{E}_g\left[g^{\delta}\right] > 0.$$

A General Tractable Framework

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- Unique properties: Effects of multiple antennas
  - Ad hoc Networks

$$\frac{\bar{p}_{n+1}}{\bar{p}_n} < 1 \text{ when } n > \frac{\mu^2}{4} - 1$$

  - Observation I:

The largest coverage improvement occurs when adding one of the first $\left\lceil \frac{\mu^2}{4} - 1 \right\rceil +$ 1 antennas, i.e., $1 \leq n^\star \leq \left\lceil \frac{\mu^2}{4} - 1 \right\rceil$.

  - Observation II:

The condition that the coverage improvement is always monotonically decreasing is given by $\frac{\mu^2}{4} - 1 < 0$, i.e., $\mu < 2$.

A General Tractable Framework

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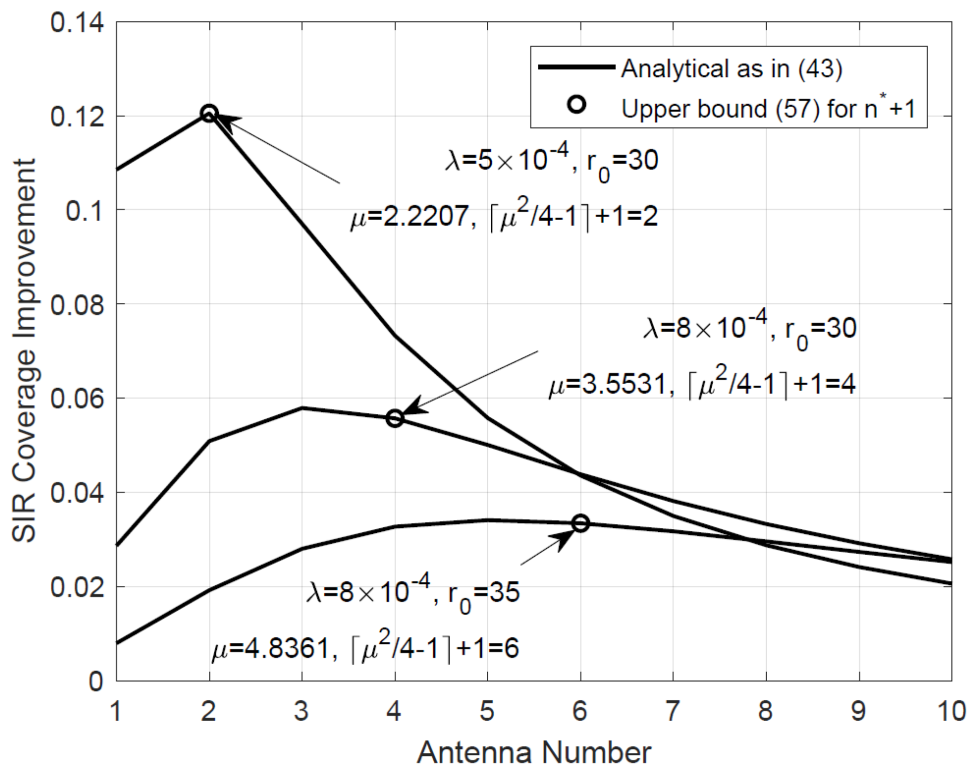
- Unique properties: Effects of multiple antennas
  - Ad hoc Networks



A General Tractable Framework

❖ Unique properties: Effects of multiple antennas

➢ General cases:

$$\bar{p}_n = \frac{(-1)^n e^{-\mu}}{n!} \sum_{k=1}^{n} s(n,k) T_k(-\mu) \delta^k$$

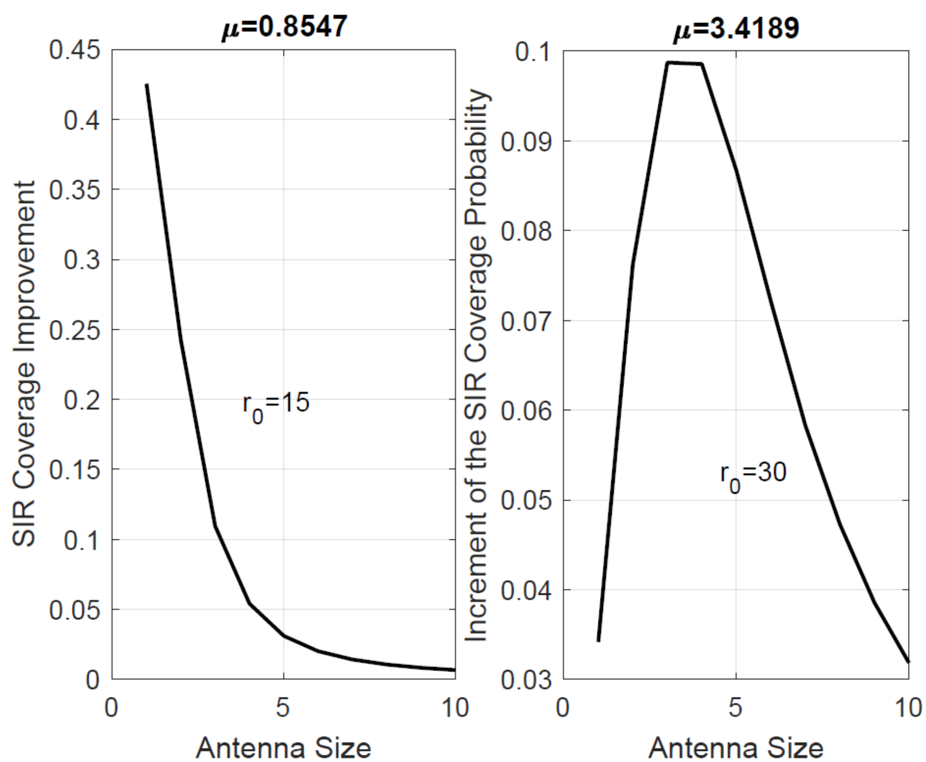
➢ Observation:

$$1 - \mu\delta > 0$$

Coverage improvement is monotonically decreasing

$$1 - \mu\delta < 0$$

Coverage improvement has a peak value



α=3

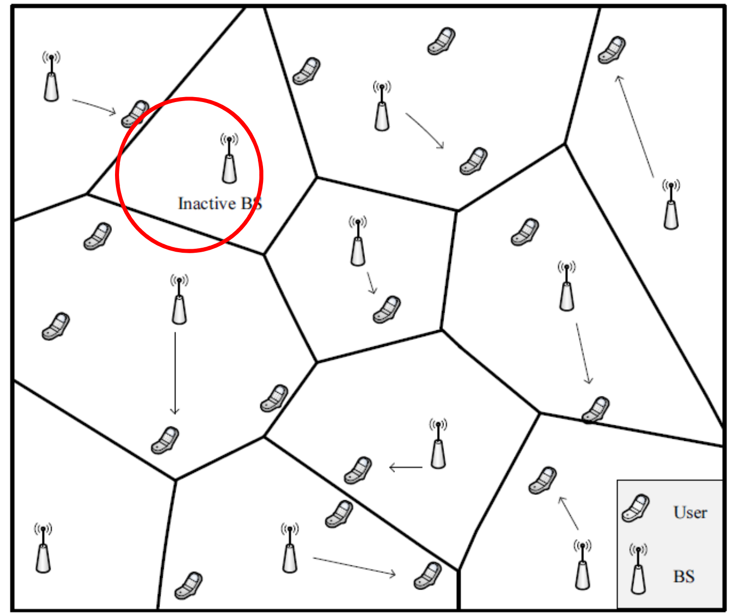
Analysis of Multi-Antenna Wireless Networks

[Ref] C. Li, **J. Zhang**, and K. B. Letaief, “Throughput and energy efficiency analysis of small cell networks with multi-antenna base stations,” *IEEE Trans. Wireless Commun.*, vol. 13, no. 5, pp. 2502-2517, May 2014.
[Ref] X. Yu, **J. Zhang**, M. Haenggi, and K. B. Letaief, “Coverage analysis for millimeter wave networks: The impact of directional antenna arrays,” *IEEE J. Select. Areas Commun.*, vol. 35, no. 7, pp. 1498-1512, Jul. 2017.

Case Study I – ASE and EE

- Motivation
  - Area spectral efficiency (ASE)

    Existing works:
    - User density large enough so all BSs are active
    - Mainly single-antenna BSs



  - Energy efficiency (EE)
    - Few result
    - Remain unclear how densification/multi-antenna affects it

Case Study I – ASE and EE

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- System model
  - BSs and users are distributed as two independent PPPs

    λ_b - BS density; λ_u - User density
  - Two kinds of BSs: Active and inactive BSs
  - BS active probability $p_a = 1 - \left(1 + \frac{\lambda_u}{3.5\lambda_b}\right)^{-3.5}$
  - N_t antennas at each BS

- Key parameters for analysis
  - Signal channel gain: $g_{x_0} \sim \mathrm{Gamma}(N_\mathrm{t}, 1)$
  - Interference: $\Phi' = P(r_0, +\infty)$ with $g_x \sim \mathrm{Gamma}(1,1)$

Case Study I – ASE and EE

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❖ The success probability of the typical user

$$p_s = \frac{1}{p_\mathrm{a}} \left\| \left[\left(k_0 + \frac{1}{p_\mathrm{a}} \right) \mathbf{I}_M - \mathbf{Q}_M \right]^{-1} \right\|_1$$

where

$$\mathbf{Q}_M = \begin{bmatrix} 0 & & & & \\ k_1 & 0 & & & \\ k_2 & k_1 & 0 & & \\ \vdots & & & \ddots & \\ k_{M-1} & k_{M-2} & \cdots & k_1 & 0 \end{bmatrix}$$

$$k_0 = \frac{\delta\tau}{1-\delta} {}_2F_1\left(1, 1-\delta; 2-\delta; -\tau\right)$$

$$k_i = \frac{\delta\tau}{i-\delta} {}_2F_1\left(i+1, i-\delta; i+1-\delta; -\tau\right)$$

[Ref] D. Commenges and M. Monsion, “Fast inversion of triangular Toeplitz matrices,” IEEE Trans. Autom. Control, vol. 29, no. 3, pp. 250–251, Mar. 1984.

Case Study I – ASE and EE

Analytical properties

Properties of ***Toeplitz matrix*** + Properties of ***L1 norm***

$$\mathbf{T}_M \triangleq \frac{1}{p_a}\left[\left(k_0 + \frac{1}{p_a}\right)\mathbf{I} - \mathbf{Q}_M\right]^{-1}$$

$$\left(p_s = \|\mathbf{T}_M\|_1\right)$$

- $\boldsymbol{T}_M$ is also a lower triangular Toeplitz matrix

$$\boldsymbol{T}_M = \begin{bmatrix} t_0 & & & & \\ t_1 & t_0 & & & \\ t_2 & t_1 & t_0 & & \\ \vdots & \ddots & \ddots & \ddots & \\ t_{M-1} & \dots & t_2 & t_1 & t_0 \end{bmatrix}$$

- $\frac{1}{1+p_a B_l} \leq \|\boldsymbol{T}_M\|_1 \leq \frac{1}{1+p_a B_u}$
- $\frac{\partial \|\mathbf{T}_M\|_1}{\partial p_a} = \frac{1}{p_a}\left(\left\|\mathbf{T}_M^2\right\|_1 - \|\mathbf{T}_M\|_1\right)$

where $B_l = k_0 - \sum_{i=1}^{M-1}\left(1 - \frac{i}{M}\right)k_i$
and $B_u = k_0 - \sum_{i=1}^{M-1} k_i$

Case Study I – ASE and EE

- Area Spectral Efficiency (ASE)
  - A tractable closed-form expression

$$R_{\mathrm{a}} = \lambda_b \left\| \left[\left(q_0 + \frac{1}{p_{\mathrm{a}}} \right) \mathbf{I} - \mathbf{Q}_M \right]^{-1} \right\|_1 \log_2 \left(1 + \hat{\gamma} \right)$$

$$q_0 = \frac{\delta \hat{\gamma}}{1-\delta} {}_2F_1 \left(1, 1-\delta; 2-\delta; -\hat{\gamma} \right) \qquad q_i = \frac{\delta \hat{\gamma}^i}{i-\delta} {}_2F_1 \left(i+1, i-\delta; i+1-\delta; -\hat{\gamma} \right)$$

  - Upper and lower bounds

$$\frac{\lambda_b R_0}{\frac{1}{p_{\mathrm{a}}} + B_l} \leq R_{\mathrm{a}} \leq \frac{\lambda_b R_0}{\frac{1}{p_{\mathrm{a}}} + B_u}.$$

Case Study I – ASE and EE

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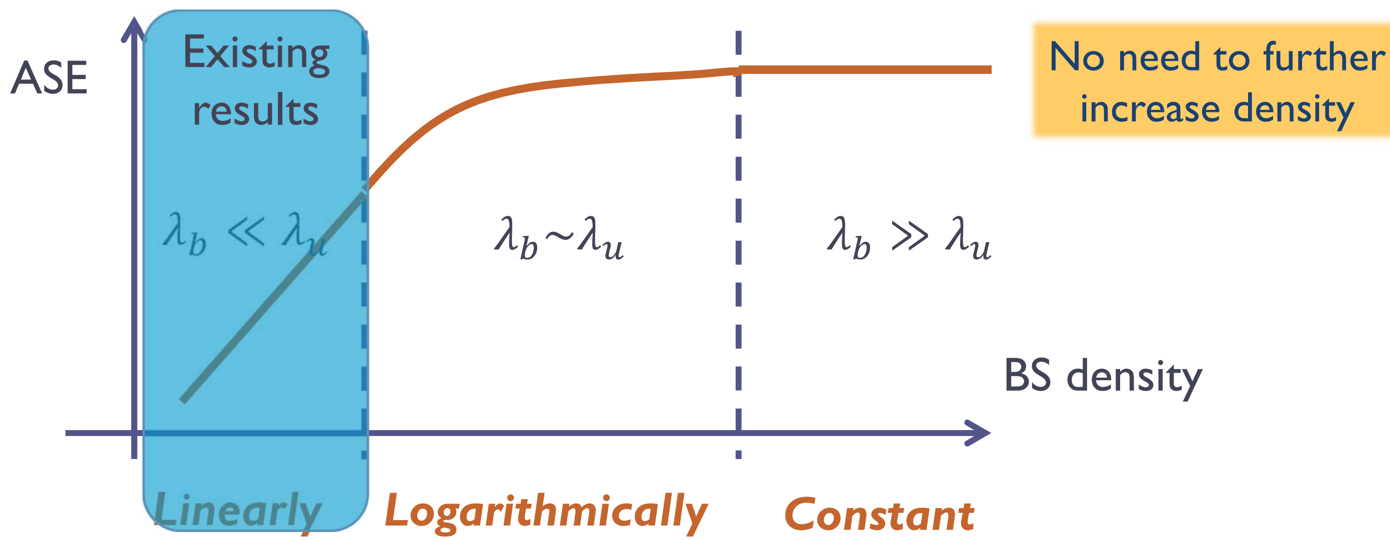
- Area Spectral Efficiency (ASE)
  - The effect of densification



  - The effect of antenna size: similar to the general case

Case Study I – ASE and EE

- Energy efficiency (EE)
  - Network energy efficiency $\eta_{\mathrm{EE}} = \frac{R_{\mathrm{a}}}{P_{\mathrm{a}}}$
  - Total power consumption of a unit area

Non-transmission power consumption

$$P_{\mathrm{a}} = \lambda_b p_{\mathrm{a}} \left(\frac{1}{\eta} P_{\mathrm{t}} + M P_{\mathrm{c}} \right) + \lambda_b P_0$$

Transmit power

Circuit power

  - ASE increases when deploying more BSs
  - Power consumption will also increase
  - How will the energy efficiency change with network densification?
  - How will the energy efficiency change with # of BS antennas?

Case Study I – ASE and EE

- Energy efficiency (EE): The effect of densification

Non-transmission power

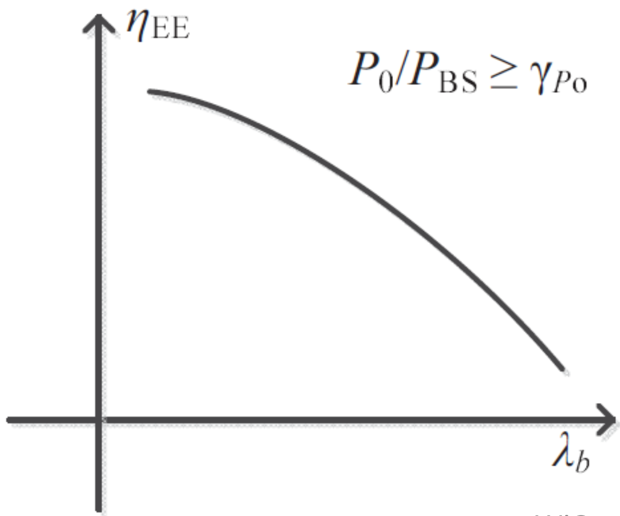
$$\frac{P_0}{P_{BS}} \geq \gamma_{P_0} \Rightarrow \frac{\partial \eta_{\mathrm{EE}}}{\partial \lambda_b} \leq 0$$

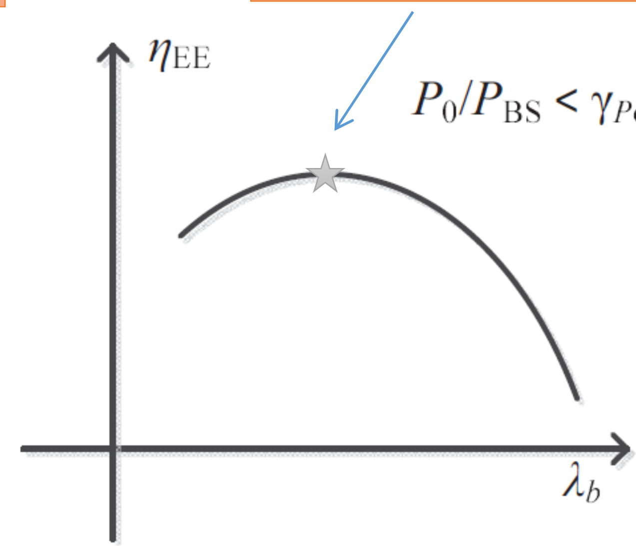
$$\frac{P_0}{P_{BS}} < \gamma_{P_0} \Rightarrow \exists \frac{\partial \eta_{\mathrm{EE}}}{\partial \lambda_b} = 0$$

where γ_{P_0} is related to M, α, and $\hat{\gamma}$.

BS power : $P_{\mathrm{BS}} = \frac{1}{\eta} P_t + M P_c + P_0$

Optimal BS density





Case Study I – ASE and EE

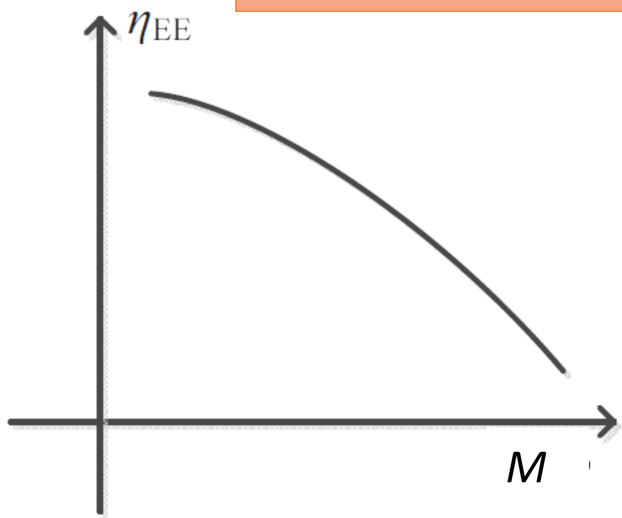
- Energy efficiency (EE): The effect of antenna size

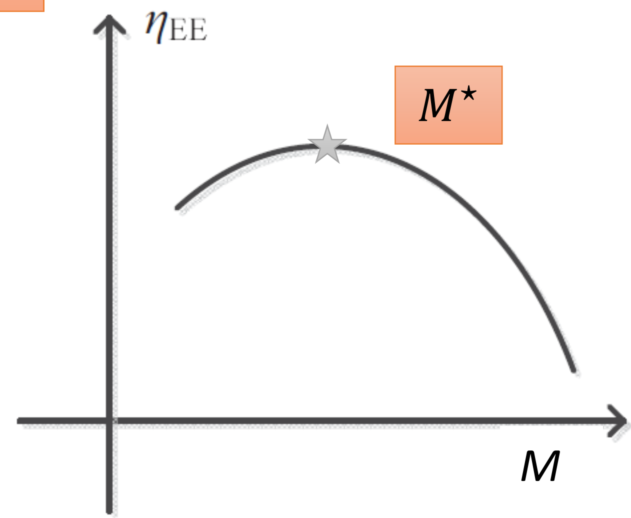
Circuit power of corresponding RF chain

$P_c \geq \gamma_{P_c} \Rightarrow \eta_{\mathrm{EE}}(1) \geq \eta_{\mathrm{EE}}(2)$

$P_c < \gamma_{P_c} \Rightarrow \exists\, M^\star > 1$

Deploying **single-antenna BS** has *higher* energy efficiency





Case Study I – ASE and EE

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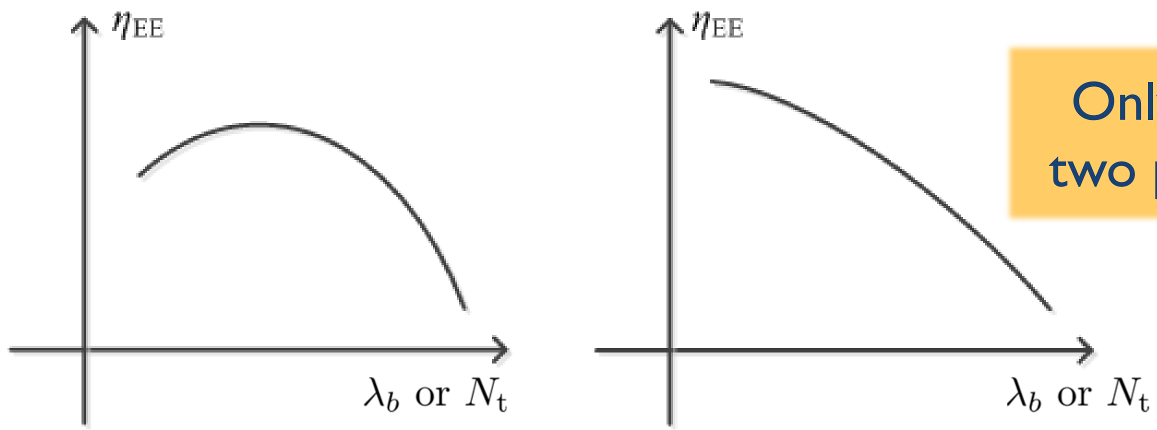
- Energy efficiency (EE)
  - Different components of the BS power consumption play important roles in the energy efficiency



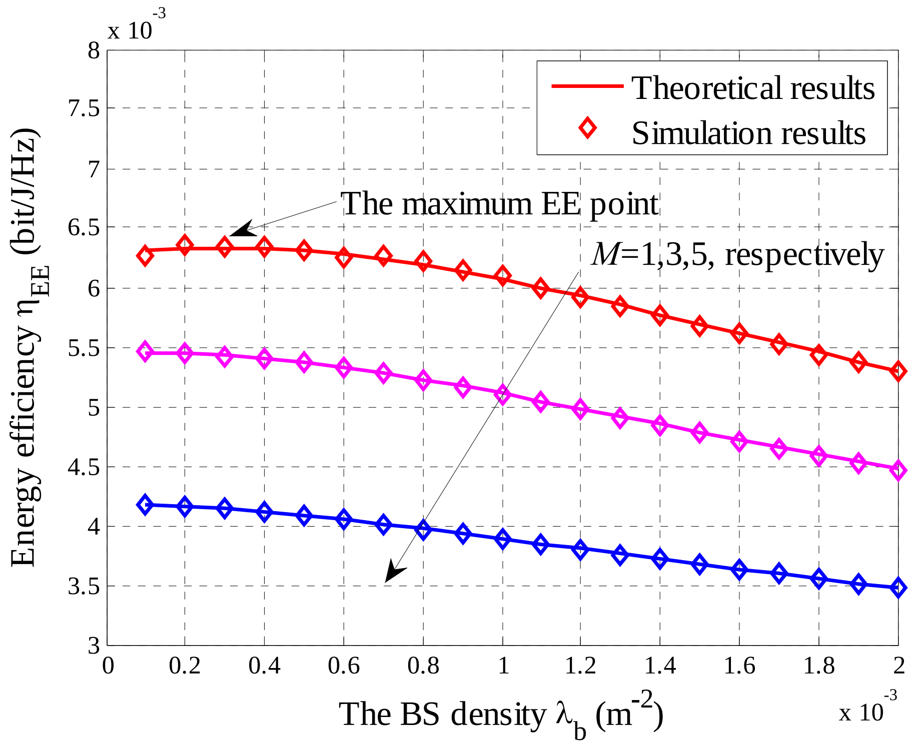
Only these two patterns

Cell densification ⇐ Non-transmission power consumption

Deploying more antennas ⇐ Circuit power consumption

Case Study I – ASE and EE

❖ Simulation result



Network Parameters

- $\lambda_u = 10^{-3}$ m^{-2}
- $\alpha = 4$
- $\hat{\gamma} = 1$
- $\sigma_n^2 = -97.5$ dbm

Power Model

- $\eta = 0.32$
- $P_t = 6.3$W
- $P_c = 35$W
- $P_0 = 34$W

➢ Based on calculation, we can get $M^{\star} = 1$ and $\lambda_b^{\star} \approx 0.3 \times 10^{-3}$ per m^2, which match the simulations

Case Study I – ASE and EE

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- Conclusions
  - A new set of analytical results for ASE and EE in multi-antenna networks
  - ASE will be increased by cell densification, but with different scaling law w.r.t. λ_b
  - EE will increase with λ_b or M only when the non-transmission power or the circuit power of a BS is less than certain thresholds

Case Study II – mm-wave Networks

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- Key factors in mm-wave networks
  - Higher frequency

    Blockage
    - Line-of-sight (LOS)
    - Non-line-of-sight (NLOS)
  - Smaller wavelength
  - Large antenna size

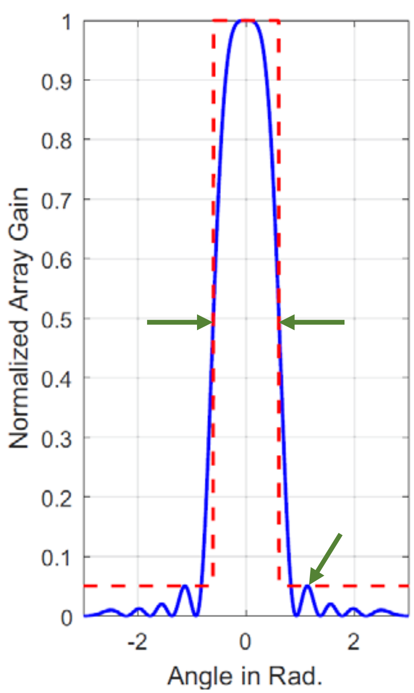
    Directional Beams

    How to model the directionality?
- Signal & interference power
  - Closely related to AoDs and AoAs

    Antenna Pattern
  - Incorporate the effect of directional antenna arrays

Case Study II – mm-wave Networks

- Antenna pattern



Huge discrepancies between the simplified and actual antenna patterns

- Flat-top: difficult to reflect the impact of antenna array accurately

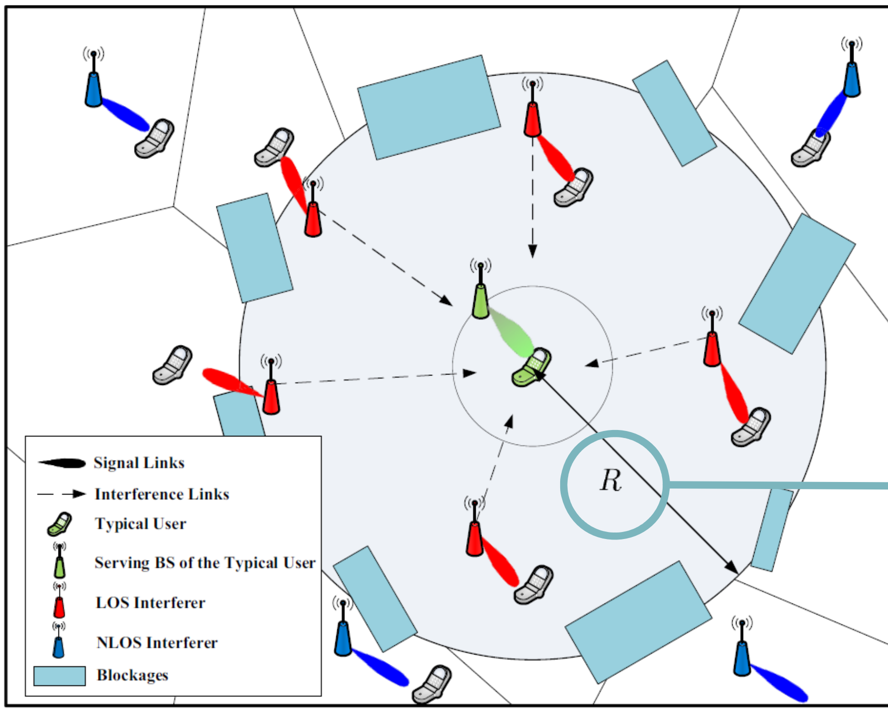
- Beamwidth
- N-th minor lobe maxima gain
- Front-back ratio
- ……

Qualitatively and inaccurately

More accurate approximation for antenna pattern!

Case Study II – mm-wave Networks

❖ Network model



- N_t-antenna BSs: $\Phi \sim \text{PPP}(\lambda_b)$
- Single antenna users
- LOS ball blockage model
- LOS interference limited

Case Study II – mm-wave Networks

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- SINR

Array gain

$$\mathrm{SINR} = \frac{P_t N_t |\alpha_0|^2 \left|\mathbf{a}_t^H(\theta_0)\mathbf{w}_0\right|^2 \beta r^{-\alpha}}{\sigma^2 + \sum_{i\in\Phi\setminus 0} P_t N_t |\alpha_i|^2 \left|\mathbf{a}_t^H(\theta_i)\mathbf{w}_i\right|^2 \beta R_i^{-\alpha}}$$

Small scale fading, no longer Rayleigh

- Key parameters for analysis

Nakagami

- Signal channel gain: $g_{x_0} \sim \mathrm{Gamma}(M, 1/M)$
- Interference: $\Phi' = P(0, R)$ with g_x

$$|\alpha_i|^2 \left|\mathbf{a}_t^H(\theta_i)\mathbf{a}_t(\phi_i)\right|^2 = |\alpha_i|^2 \frac{\sin^2\left[\frac{N_t}{2} kd(\cos\theta_i - \cos\phi_i)\right]}{N_t^2 \sin^2\left[\frac{1}{2} kd(\cos\theta_i - \cos\phi_i)\right]}$$

Case Study II – mm-wave Networks

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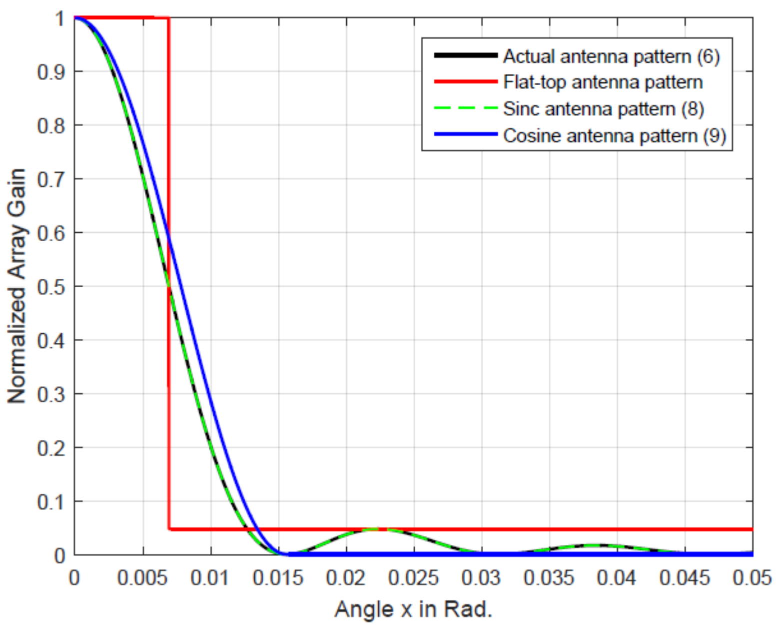
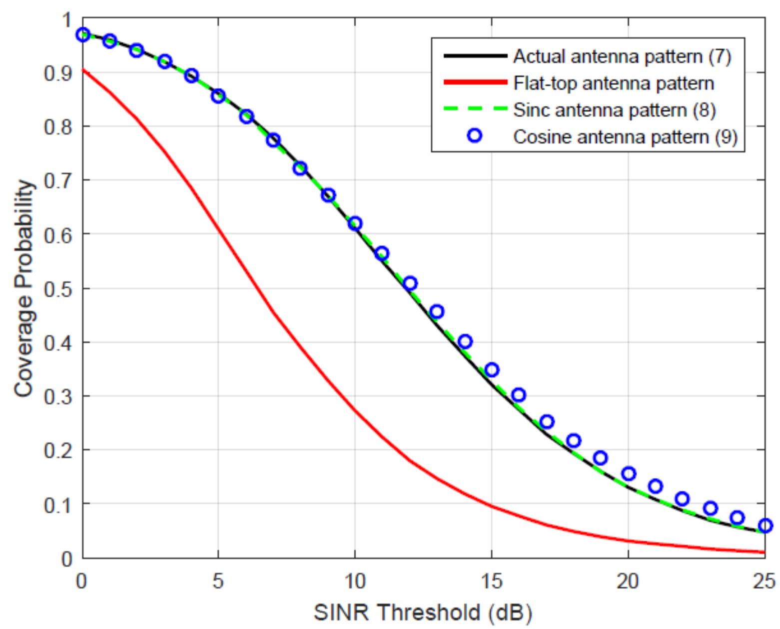
❖ Approximated antenna pattern

$$G_{\cos}(x) = \begin{cases} \cos^2\left(\frac{\pi N_{\mathrm{t}}}{2} x\right) & |x| \leq \frac{1}{N_{\mathrm{t}}}, \\ 0 & \text{otherwise}, \end{cases}$$





Case Study II – mm-wave Networks

- Impact of directional antenna arrays

$$p_{\rm c}^{\cos}(\tau) \geq \left(1 - e^{-\pi\lambda_{\rm b}R^2}\right)\left\|\exp\left\{\frac{1}{N_{\rm t}(1-e^{-\pi\lambda_{\rm b}R^2})}\mathbf{Q}_M\right\}\right\|_1$$
$$= \left(1 - e^{-\pi\lambda_{\rm b}R^2}\right)e^{\beta_0 t}\left(1 + \sum_{n=1}^{M-1}\beta_n t^n\right)$$

$$t = \frac{1}{N_{\rm t}} \qquad \beta_n = \begin{cases} \dfrac{q_0}{1-e^{-\pi\lambda_{\rm b}R^2}} & n = 0, \\ \dfrac{\|(\mathbf{Q}_M - q_0\mathbf{I}_M)^n\|_1}{n!\left(1-e^{-\pi\lambda_{\rm b}R^2}\right)} & n \geq 1. \end{cases}$$

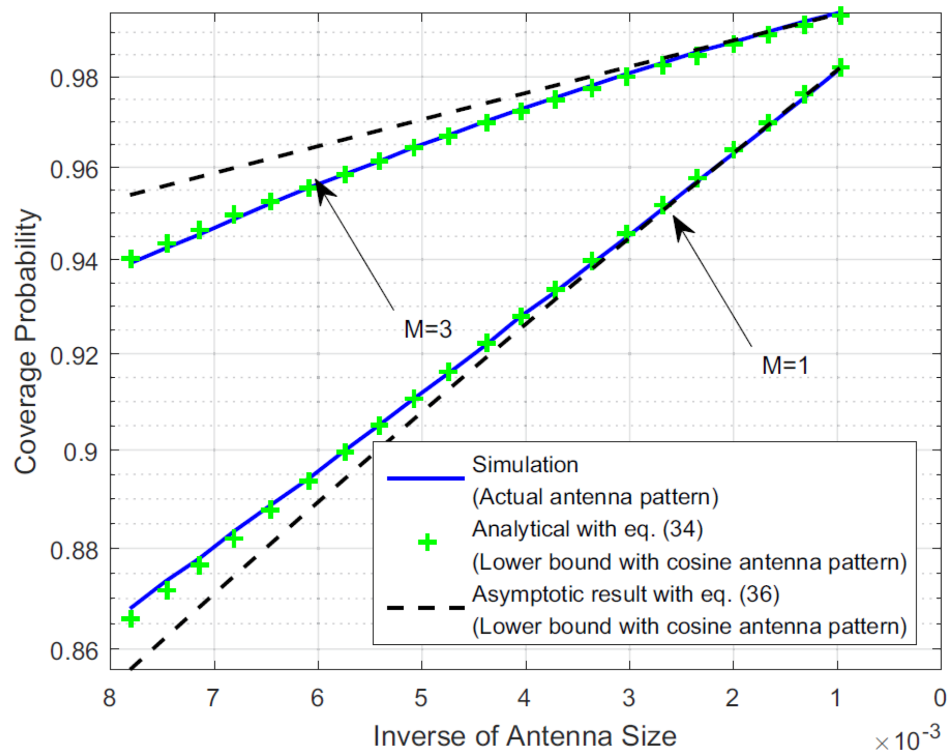
- Asymptotic result (Outage Probability)

$$\tilde{p}_{\rm o}^{\cos}(t) \sim \frac{\mu}{N_{\rm t}} + e^{-\pi\lambda_{\rm b}R^2} \qquad \mu = -\sum_{n=0}^{M-1} d_n > 0$$

  - Inversely proportional to the array size

Case Study II – mm-wave Networks

❖ Impact of directional antenna arrays



Case Study II – mm-wave Networks

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- Conclusions
  - An accurate approximation of the directional antenna pattern is desired
  - The coverage is a monotone increasing function of the array size, which is the product of an exponential and a polynomial function
  - The asymptotic outage probability is inversely proportional to the array size

Optimization of Multi-Antenna Wireless Networks

[Ref] C. Li, **J. Zhang**, J. G. Andrews, and K. B. Letaief, "Success probability and area spectral efficiency in multiuser MIMO HetNets," *IEEE Trans. Commun.*, vol. 64, no. 4, pp. 1544-1556, Apr. 2016.

[Ref] C. Li, **J. Zhang**, M. Haenggi, and K. B. Letaief, "User-centric intercell interference nulling for downlink small cell networks," *IEEE Trans. Commun.*, vol. 63, no. 4, pp. 1419-1431, Apr. 2015.

[Ref] X. Yu, C. Li, **J. Zhang**, K. B. Letaief, "A tractable framework for performance analysis of dense multi-antenna networks," in *Proc. IEEE Int. Conf. Commun. (ICC),* Paris, France, May 2017.

Case Study III – ASE vs. Link Reliability

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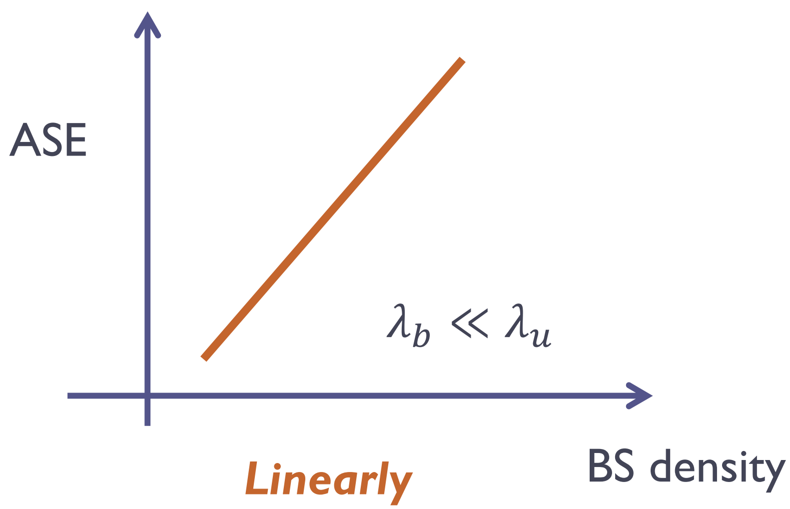
- SIR Invariance
  - Recall that in single-tier networks

ASE will increase linearly by deploying more BSs if $\lambda_b \ll \lambda_u$



For other networks?

Case Study III – ASE vs. Link Reliability

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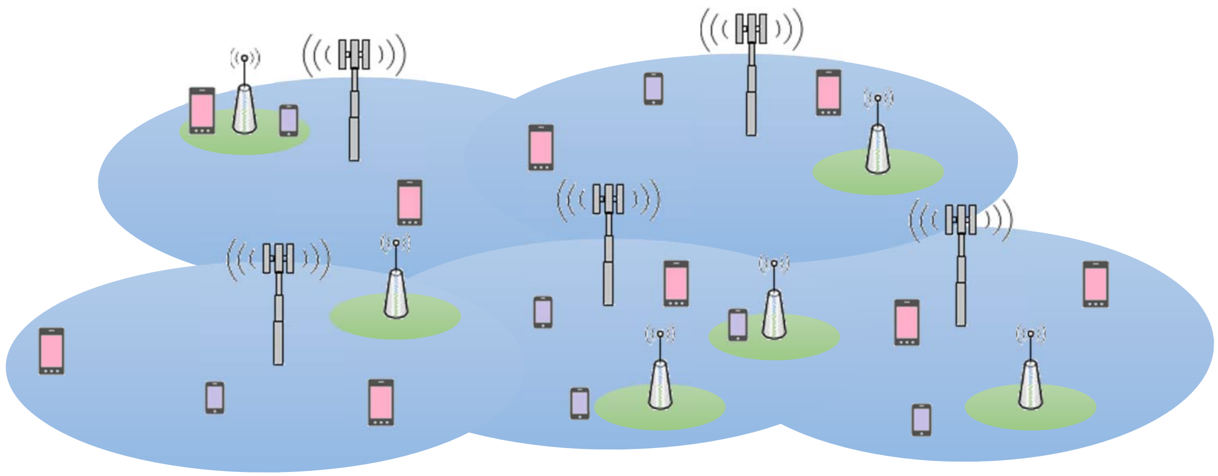
❖ A general HetNet



Different types of BSs:

- Densities,
- TX powers,
- User association rules,
- # of TX antennas,
- Serve different # of users (SDMA)

→ SISO HetNets

→ **Multiuser MIMO HetNets**

Case Study III – ASE vs. Link Reliability

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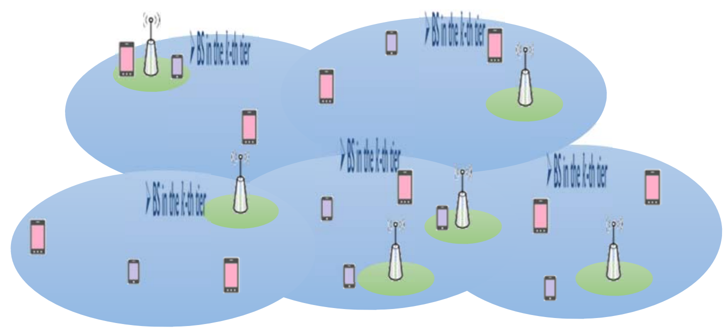
- System model
  - A K-tier Downlink HetNet
  - BS in the k-th tier

    Density: λ_k
    TX power: P_k
    # of antennas: M_k
    # of served users (SDMA): U_k
    User association bias: B_k



  - Users: Single receive antenna and will be associated with the BS in the k-th tier if

$$k = \arg\max_{j \in \mathcal{K}} P_j B_j r_j^{-\alpha}$$

  - Channel: Rayleigh fading channel, and universal frequency reuse

Case Study III – ASE vs. Link Reliability

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- Key parameters for analysis
  - Signal channel gain: $g_{x_0} \sim \mathrm{Gamma}(M_k - U_k + 1, 1)$
  - Interference: $\Phi_j' = P(r_j, +\infty)$ with $g_{x,j} \sim \mathrm{Gamma}\left(U_j, 1\right)$

- Success probability
  - Tractable closed-form expression

$$p_s(\gamma) = \sum_{k=1}^{K} \left\| \mathbf{Q}_{M_k - U_k + 1}^{-1} \right\|_1$$

$$q_{k,i} = \frac{1}{P_k^\delta B_k^\delta} \sum_{j=1}^{K} \lambda_j P_j^\delta B_j^\delta \frac{\Gamma(U_j+i)}{\Gamma(U_j)\Gamma(i+1)} \frac{\delta}{i-\delta} \left(\frac{U_k B_k}{U_j B_j}\tau\right)^i \times {}_2F_1\left(i-\delta, U_j+i; i+1-\delta; -\frac{U_k B_k}{U_j B_j}\tau\right)$$

  - Enables more sophisticated analysis and optimization

Case Study III – ASE vs. Link Reliability

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- Asymptotic expression of p_s as SIR threshold $\hat{\gamma} \to \infty$

$$p_{\mathrm{s}} \sim \tau^{-\delta} \mathrm{sinc}\left(\delta\right) \frac{\sum_{k=1}^{K} \lambda_k \left(\frac{P_k}{U_k}\right)^{\delta} \frac{\Gamma(D_k+\delta)}{\Gamma(D_k)}}{\sum_{j=1}^{K} \lambda_j \left(\frac{P_j}{U_j}\right)^{\delta} \frac{\Gamma(U_j+\delta)}{\Gamma(U_j)}} = \tau^{-\delta} \mathrm{sinc}\left(\delta\right) \frac{\mathbf{c}^T \boldsymbol{\lambda}}{\mathbf{d}^T \boldsymbol{\lambda}}$$

- Key properties
  - p_s is monotonic (either increase or decrease) w.r.t. BS density λ_k

    No SIR invariance property!

  - The maximum p_s is obtained by activating only one tier of BSs

$$p_{\mathrm{s}}^{\max} = p_{\mathrm{s}}\left(k\right) \quad \text{for } k = \arg\max_{j} \frac{\Gamma\left(D_j+\delta\right)/\Gamma\left(D_j\right)}{\Gamma\left(U_j+\delta\right)/\Gamma\left(U_j\right)}$$

- There exists a trade-off between ASE and link reliability

Case Study III – ASE vs. Link Reliability

- ASE vs. link reliability trade-off
  - Special Case: $U_k = U$

Link Reliability	ASE
The maximum p_s is obtained by activating only one tier of BSs which have the largest # of antennas	The maximum ASE is achieved by activating all the BSs.

$$p_{\mathrm{s}} = \frac{\sum_{k=1}^{K} \left(\frac{P_k}{U}\right)^{\delta} p_{\mathrm{s}}(k)\,\lambda_k}{\sum_{k=1}^{K} \left(\frac{P_k}{U}\right)^{\delta} \lambda_k}$$

$$\mathrm{ASE} = U \log_2(1+\tau) \sum_{k=1}^{K} \lambda_k p_{\mathrm{s}}(k)$$

$$\begin{aligned} \mathcal{P}_o : \underset{\{\lambda_k\}}{\text{maximize}} \quad & \mathrm{ASE} \\ \text{subject to} \quad & p_{\mathrm{s}} \geq \Theta, \\ & 0 \leq \lambda_k \leq \lambda_k^{\max}, \forall k \in \mathcal{K}, \end{aligned}$$

(LP problem)

Case Study III – ASE vs. Link Reliability

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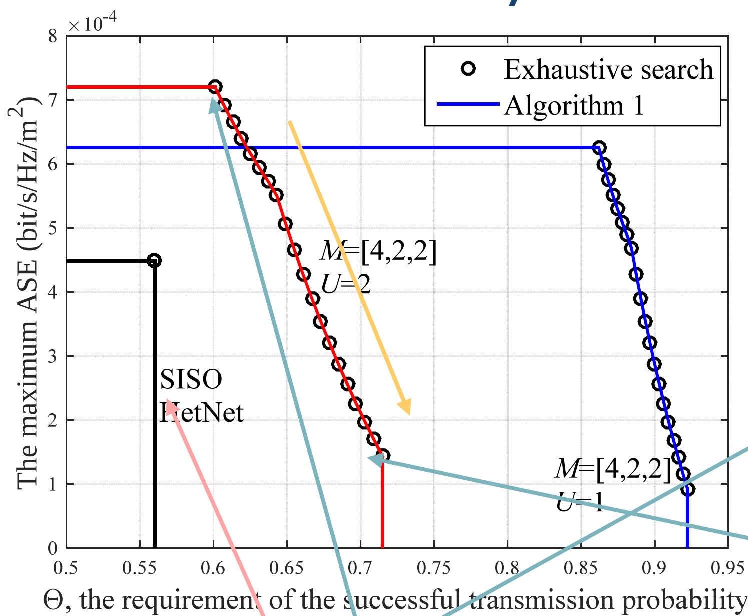
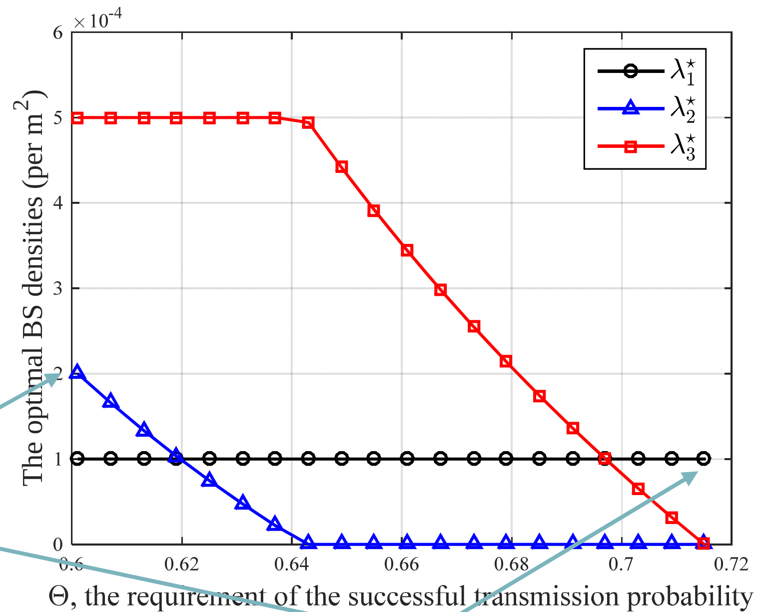
❖ ASE vs. link reliability trade-off





The maximum ASE is obtained by turning on all the BSs

The maximum p_s of the network is achieved by only turning on the first tier BSs.

There is no such a tradeoff in SISO HetNets, due to the invariance property

By turning off BSs with small # of antennas, the link reliability of each transmission link will increase, but the ASE will decrease

Case Study III – ASE vs. Link Reliability

- ASE vs. link reliability trade-off
  - General case

Link Reliability	ASE
$p_{\mathrm{s}} \sim \hat{\gamma}^{-\delta}\mathrm{sinc}\left(\delta\right)\frac{\mathbf{c}^T\boldsymbol{\lambda}}{\mathbf{d}^T\boldsymbol{\lambda}}$	$\mathrm{ASE} \sim \hat{\gamma}^{-\delta}\mathrm{sinc}\left(\delta\right)\log_2\left(1+\tau\right)\frac{\left(\mathbf{c}_1^T\boldsymbol{\lambda}\right)\left(\mathbf{c}_2^T\boldsymbol{\lambda}\right)}{\mathbf{d}^T\boldsymbol{\lambda}}$

$$
\begin{aligned}
\mathcal{P}_1 : \underset{\boldsymbol{\lambda}}{\text{maximize}} \quad & \frac{(\mathbf{c}_1^T\boldsymbol{\lambda})(\mathbf{c}_2^T\boldsymbol{\lambda})}{\mathbf{d}^T\boldsymbol{\lambda}} \\
\text{subject to} \quad & \tau^{-\delta}\mathrm{sinc}\left(\delta\right)\frac{\mathbf{c}^T\boldsymbol{\lambda}}{\mathbf{d}^T\boldsymbol{\lambda}} \geq \Theta, \\
& 0 \leq \lambda_k \leq \lambda_k^{\max}, k \in \mathcal{K}.
\end{aligned}
$$

(Non-convex problem)

A sub-optimal algorithm via sequential convex programming

Case Study III – ASE vs. Link Reliability

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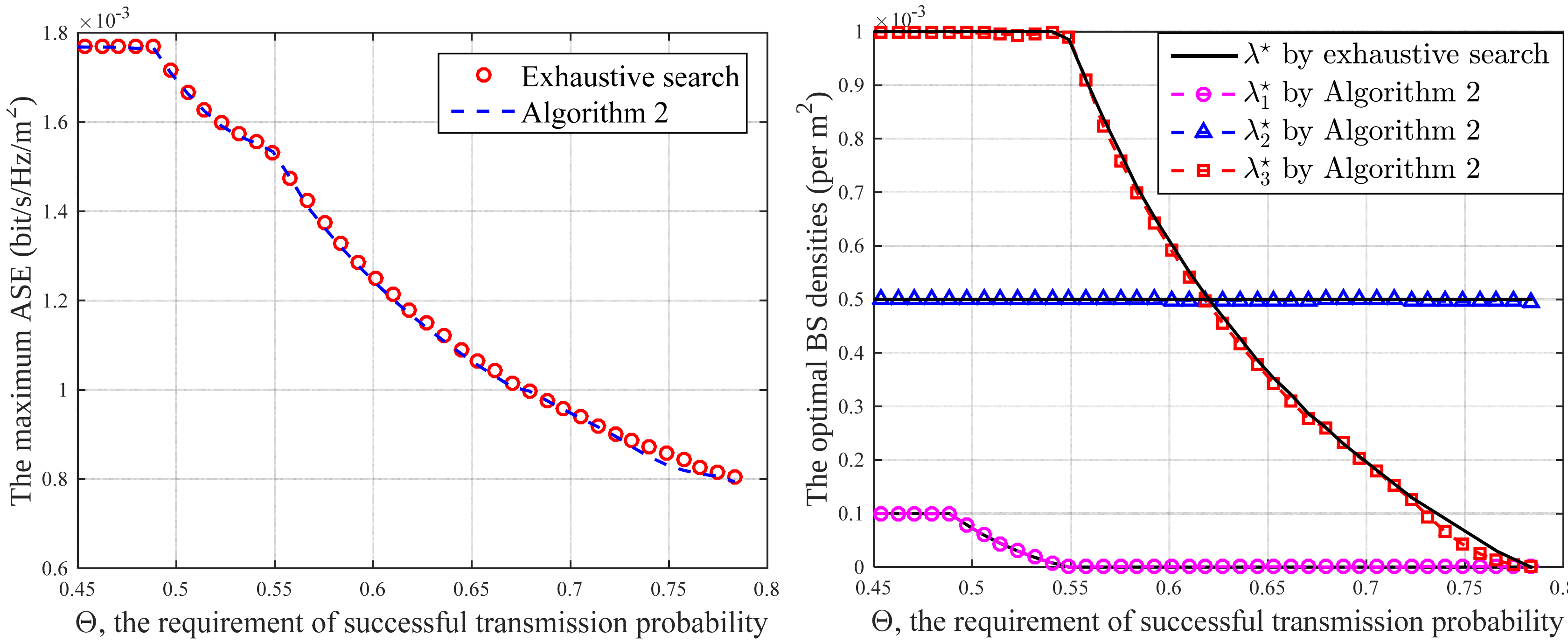
- ASE vs. link reliability trade-off



$$[M_1, M_2, M_3] = [8, 4, 1],\ [U_1, U_2, U_3] = [4, 1, 1]$$

Case Study III – ASE vs. Link Reliability

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- Conclusions
  - Provided tractable expressions of the success probability in general MIMO HetNets
  - SIR invariance property no longer holds in MIMO HetNets, and the maximum success probability is achieved by activating only one tier of BSs
  - There is a unique tradeoff between the ASE and link reliability in MIMO HetNets. Both of them need to be considered in designing or densifying the network

Case Study IV – Interference Nulling

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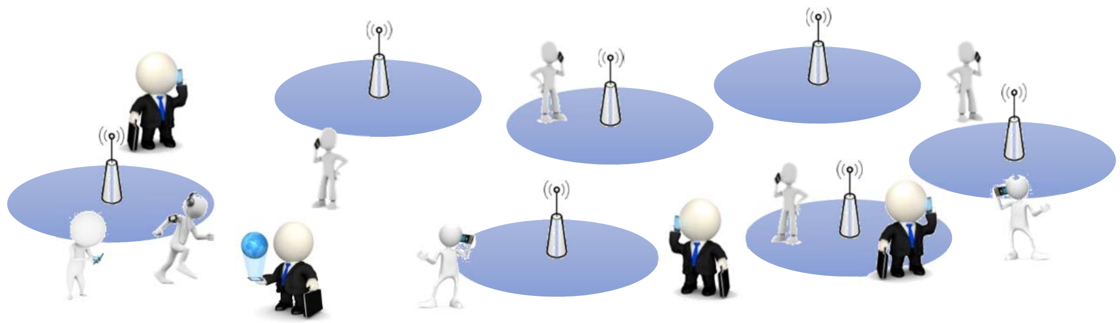
❖ Interference is the bottleneck in celular networks



SIR threshold	0dB		10dB
# of antennas per BS	$M = 1$	$M = 8$ (MRT)	
Outage probability	>40%	<1%	>40%

- **Q:** Can *interference coordination* improve the performance?

Case Study IV – Interference Nulling

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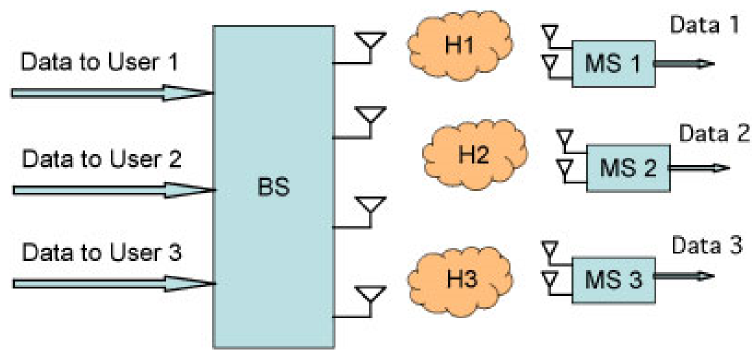
- Coordination Design: New perspective
  - Conventional interference coordination design

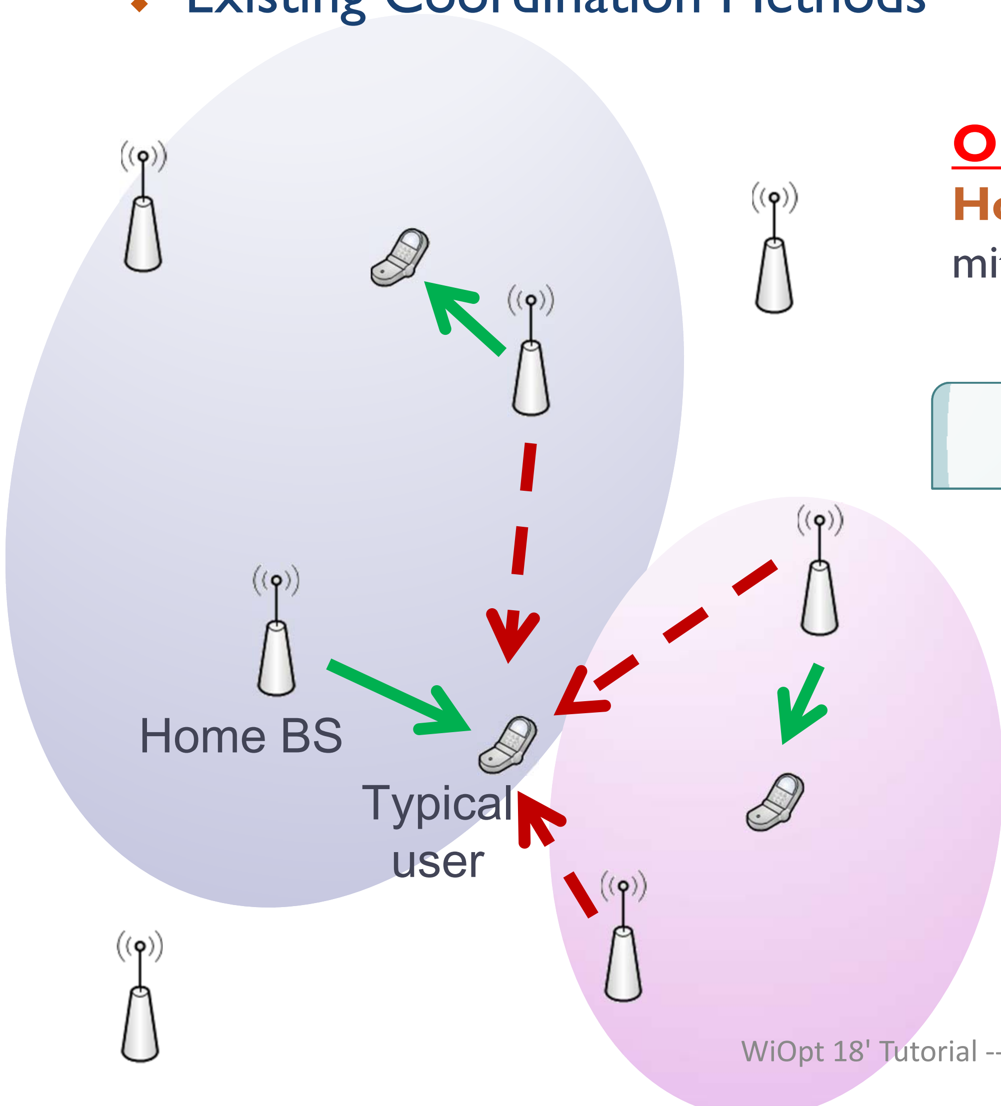


  - However, when considering from the large-scale network perspective

How many BSs, and **which BSs** should mitigate interference for a user?

Case Study IV – Interference Nulling

❖ Existing Coordination Methods



Our concern:
How many BSs, and **which BSs** should mitigate interference for a user?

Disjoint BS clustering
[S. Akoum, and R. W. Heath, Apr. 2013]

Cluster-edge users still suffer strong inter-cluster interference

Case Study IV – Interference Nulling

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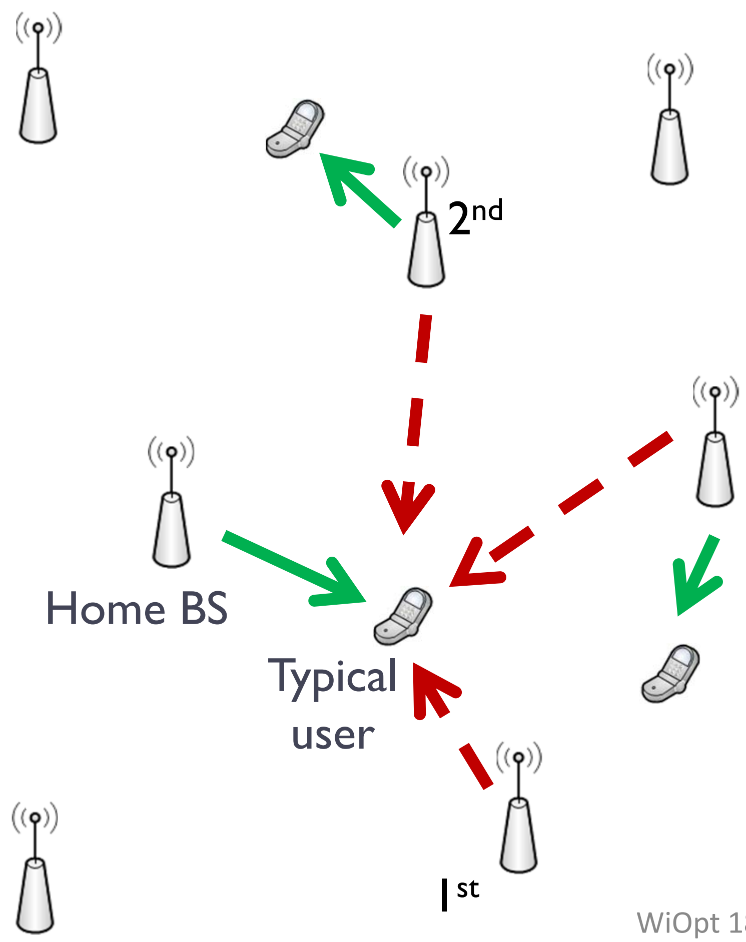
Existing Coordination Methods



Our concern:
How many BSs, and **which BSs** should mitigate interference for a user?

Disjoint BS clustering
[S. Akoum, and R. W. Heath, Apr. 2013]

Cluster-edge users still suffer strong inter-cluster interference

User-centric interference nulling with a fixed number of coordination BSs

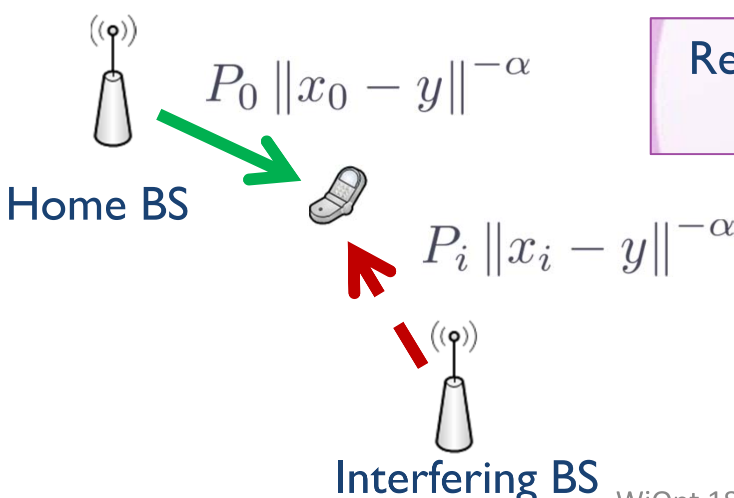
Different users have different # of dominant interferers.

Case Study IV – Interference Nulling

- A novel approach
  - A novel method to effectively determine the dominant interfering BSs for each user – **A user-centric approach**

$$\frac{\text{Received interference power from the } i\text{th BS}}{\text{Received signal power from the home BS}} \geq \text{Threshold}$$

Strong interfering source!

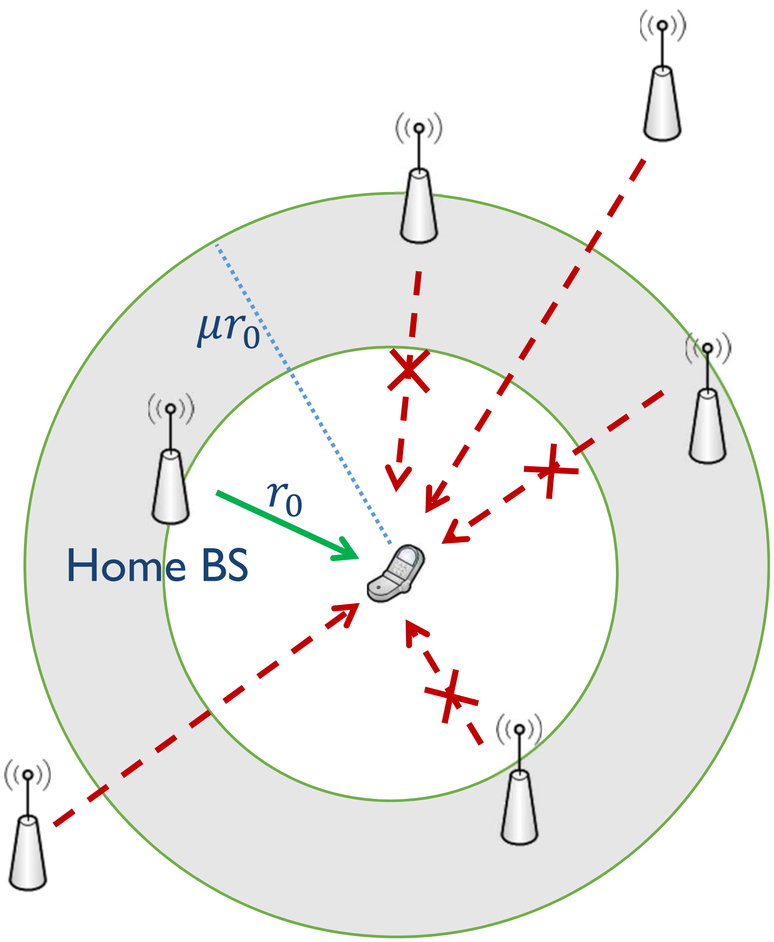


The coordinating BSs are determined by

$$\left\{ x_i : \frac{P_i \|x_i - y\|^{-\alpha}}{P_0 \|x_0 - y\|^{-\alpha}} \geq \theta \right\}$$

Case Study IV – Interference Nulling

❖ System model



- BSs and users are two independent PPPs with densities λ_b and λ_u.
- Each BS has M antennas, and uses transmit power P_t.
- Set $\mu \geq 1$ as the coordination range coefficient.

$$\{x_i : \quad \|x_i\| \leq \mu r_0\}$$

Increase μ:

- More nearby interference will be mitigated
- Each BS has less DoFs to serve its own user

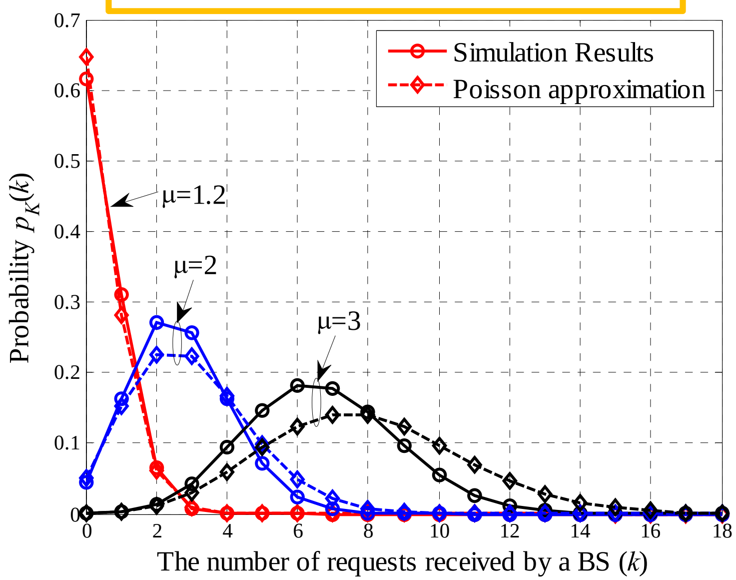
Case Study IV – Interference Nulling

❖ Success probability

➢ The success probability of the typical user is given by

$$p_s = \mathbb{E}_{K_{x_0}}\left[\mathbb{P}\left(\mathrm{SIR}_k \geq \tau \mid K_{x_0} = k\right)\right]$$

PMF of the # of requests received by a BS



$$p_{\mathrm{s}}(k) \approx \left\| \left[\mathbf{I}_l + p_{\mathrm{a}} \mathbf{Q}_l\right]^{-1} \right\|_1$$

- Toeplitz matrix
- Determined by μ and τ

$$l = \max\left(M - k, 1\right)$$

Case Study IV – Interference Nulling

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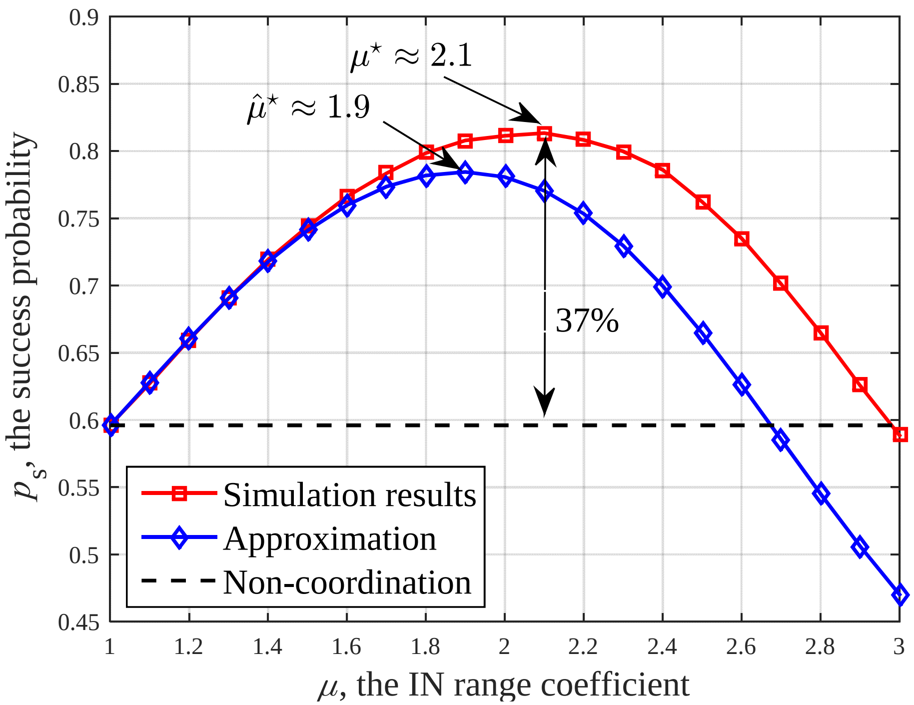
Effect of μ



- # of antennas at each BS: M=8
- BS density: $\lambda_b = 10^{-3}$ per m^2
- User density: $\lambda_u = 10^{-2}$ per m^2
- Path loss exponent: $\alpha = 4$
- SINR threshold: τ =10 dB

✓ Significant performance gain

✓ Approximation is more accurate with a small μ

✓ The approximated optimal μ is closed to the optimal μ in simulation.

Case Study IV – Interference Nulling

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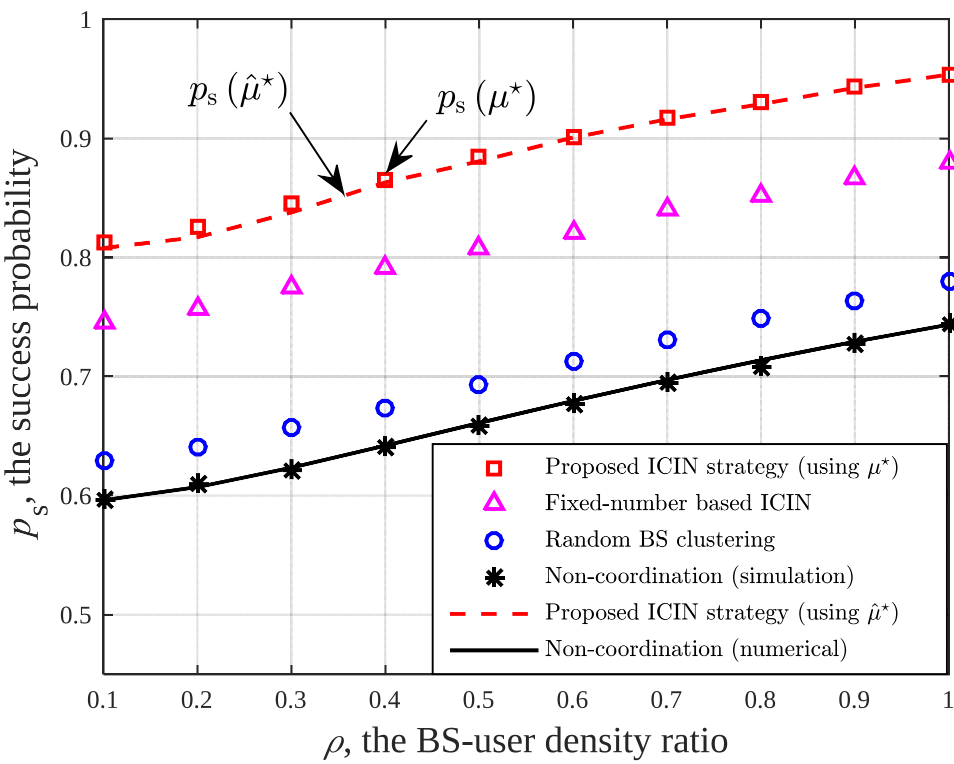
❖ Performance comparison



- # of antennas at each BS: M=8
- BS density: $\lambda_b = 10^{-3}$ per m^2
- Path loss exponent: $\alpha = 4$
- SINR threshold: $\hat{\gamma}$=10 dB

The proposed ICIN (with the optimal μ)

ICIN with fixed number of requests sent by each user (using the optimal N)

BS clustering (with the optimal cluster size)

Non-coordination strategy ($\mu = 1$)

Case Study IV – Interference Nulling

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- Conclusions
  - Intercell interference is the key factor which limits the network performance
  - Our analytical results can help to design the effective interference coordination scheme, which is impossible to be done by simulation
  - When using interference coordination, it is critical to identify and mitigate the dominant interference

Case Study V – PHY-Layer Security

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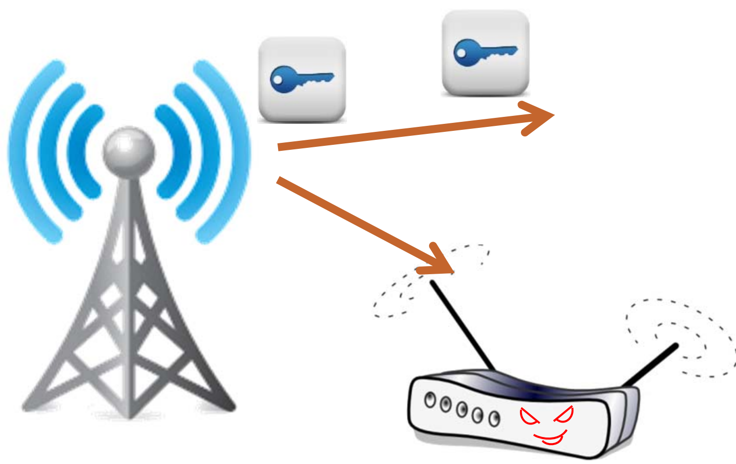
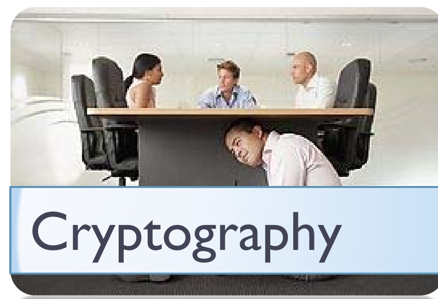
❖ Broadcast nature of wireless medium



Eavesdropper



Cryptography

- e.g., symmetric data encryption/decryption algorithm
- Tradeoff between security & computational power

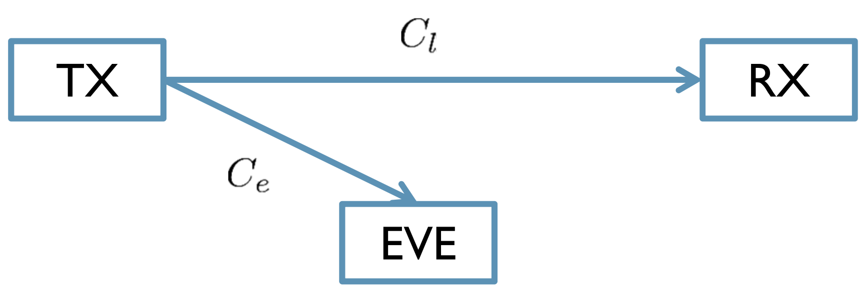
Physical-layer security

- Additional layer of security
- Low cost
- No key required

A. D. Wyner, 1975

Case Study V – PHY-Layer Security

- Secrecy capacity
  - The secrecy capacity corresponds to the difference between the legitimate user's channel capacity and the eavesdropper's (EVE's)



$C_l - C_e$ - secrecy capacity

How much information can be transmitted without being intercepted.

Case Study V – PHY-Layer Security

- System model
  - Transmitters

    Distributed as a PPP with density λ_l
    Each has one single-antenna receiver with distance d_0
    Each has M antennas

  - Eavesdroppers

    Distributed as a PPP with density λ_e
    Each has single antenna
    They are passive and do not collude

- A novel approach to enhance the secrecy capacity

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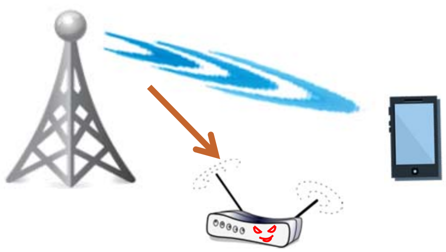
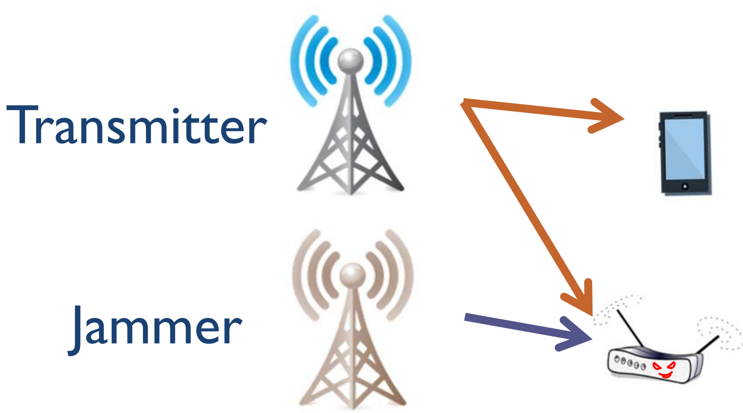
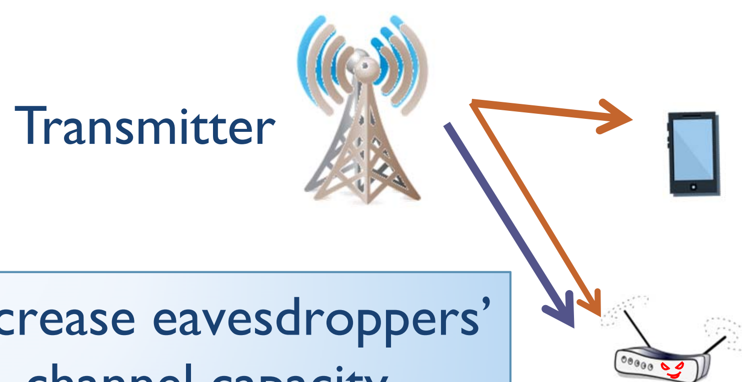
- Conventional secrecy capacity enhancement
  - MIMO technique – Beamforming



Increase legitimate channel capacity

  - Cooperative jamming or jamming-aided transmission

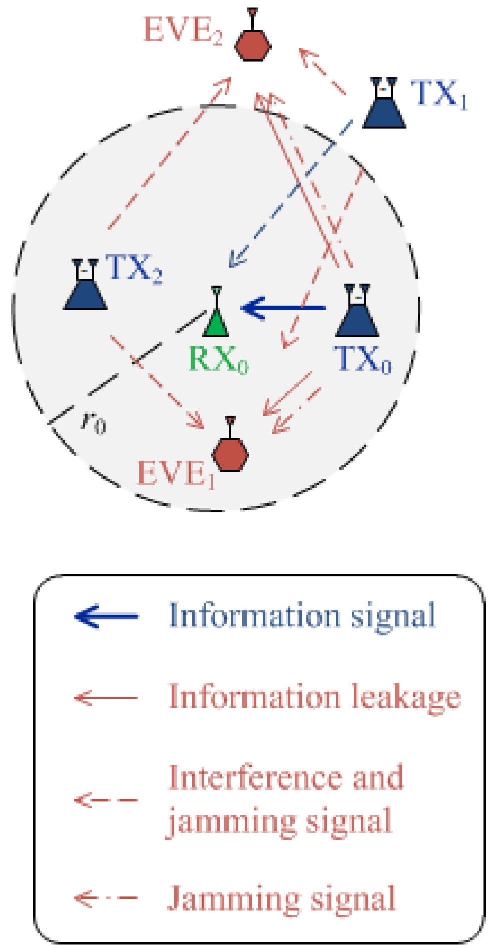




Decrease eavesdroppers' channel capacity

Case Study V – PHY-Layer Security

- Proposed joint jamming and interference nulling
  - Each receiver sends interference nulling requests to the transmitters within distance r_0
  - The transmitter receives K_x requests, and it will null interference for them.
  - The transmitter sends jamming noise to the channels orthogonal to the K_x receiver as well as its own receiver.



Case Study V – PHY-Layer Security

- Connection outage probability $p_{\mathrm{co}} = \mathbb{P}\left(\mathrm{SIR}_l \leq \hat{\gamma}_l\right)$
  - Outage happens if the typical user cannot decode the information
- Key parameters for analysis
  - Signal channel gain: $g_{x_0} \sim \mathrm{Gamma}(N_{x_0}, 1)$
  - $\Phi'_{\mathrm{out}} = P(r_0, +\infty)$ with $g_{x,\mathrm{out}} \sim \mathrm{Gamma}(N_x, 1)$
    $\Phi'_{\mathrm{in}} = P(0, r_0)$ with $g_{x,\mathrm{in}} \sim \mathrm{Gamma}(1,1)$

$$p_{\mathrm{co}} = 1 - \sum_{N_{x_0}=1}^{N_{\mathrm{t}}} p_N\left(N_{x_0}\right) p_{\mathrm{co}}\left(N_{x_0}\right)$$

$$p_N(n) = \begin{cases} \frac{\left(\pi d_0^2 \lambda_{\mathrm{t}}\right)^{N_{\mathrm{t}}-n}}{(N_{\mathrm{t}}-n)!} e^{-\pi d_0^2 \lambda_{\mathrm{t}}} & n = 2, 3, \cdots, N_{\mathrm{t}} \\ 1 - \sum_{i=2}^{N_{\mathrm{t}}} p_N(i) & n = 1 \end{cases}$$

$$p_{\mathrm{co}}\left(N_{x_0}\right) = 1 - \left\| e^{-\pi \lambda_l r_0^2 \left[\mathbf{Q}_{N_{x_0}} - (1-\varepsilon)\mathbf{I}_{N_{x_0}}\right]} \right\|_1$$

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- Secrecy outage probability $p_{\mathrm{so}} = 1 - \mathbb{E}_{\Psi_l}\left[\mathbb{E}_{\Psi_e}\left[\prod_{z\in\Psi_e}\mathbb{P}\left(\mathrm{SIR}_{e,z} \le \hat{\gamma}_e \mid \Psi_l\right)\right]\right]$
  - Outage happens if at least one EVE can decode the information
- Key parameters for analysis
  - Signal channel gain: $g_{x_0} \sim \mathrm{Gamma}(1,1)$
  - Point x_0 with $g_{x,\mathrm{out}} \sim \mathrm{Gamma}(N_{x_0} - 1,1)$

    $\Phi' = P(0,+\infty)\backslash\{x_0\}$ with $g_x \sim \mathrm{Gamma}(N_x, 1)$

$$p_{\mathrm{so}}^{\mathrm{UB}} = 1 - \sum_{N_{x_0}=1}^{M} p_N\left(N_{x_0}\right)\exp\left\{-\frac{\lambda_e}{\lambda_l}\frac{\left(1+\hat{\gamma}_e\right)^{1-N_{x_0}}\hat{\gamma}_e^{-\delta}N_{x_0}^{-\delta}}{\Gamma\left(1-\delta\right)\sum_{N=1}^{M}p_N\left(N\right)\frac{\Gamma(N+\delta)}{\Gamma(N)N^{\delta}}}\right\}$$

$$p_{\mathrm{so}}^{\mathrm{LB}} = \sum_{N_{x_0}=1}^{M} p_N\left(N_{x_0}\right)\frac{\left(1+\hat{\gamma}_e\right)^{1-N_{x_0}}}{1+\frac{\lambda_l}{\lambda_e}\Gamma\left(1-\delta\right)\hat{\gamma}_e^{\delta}N_{x_0}^{\delta}\sum_{N=1}^{M}p_N\left(N\right)\frac{\Gamma(N+\delta)}{\Gamma(N)N^{\delta}}}$$

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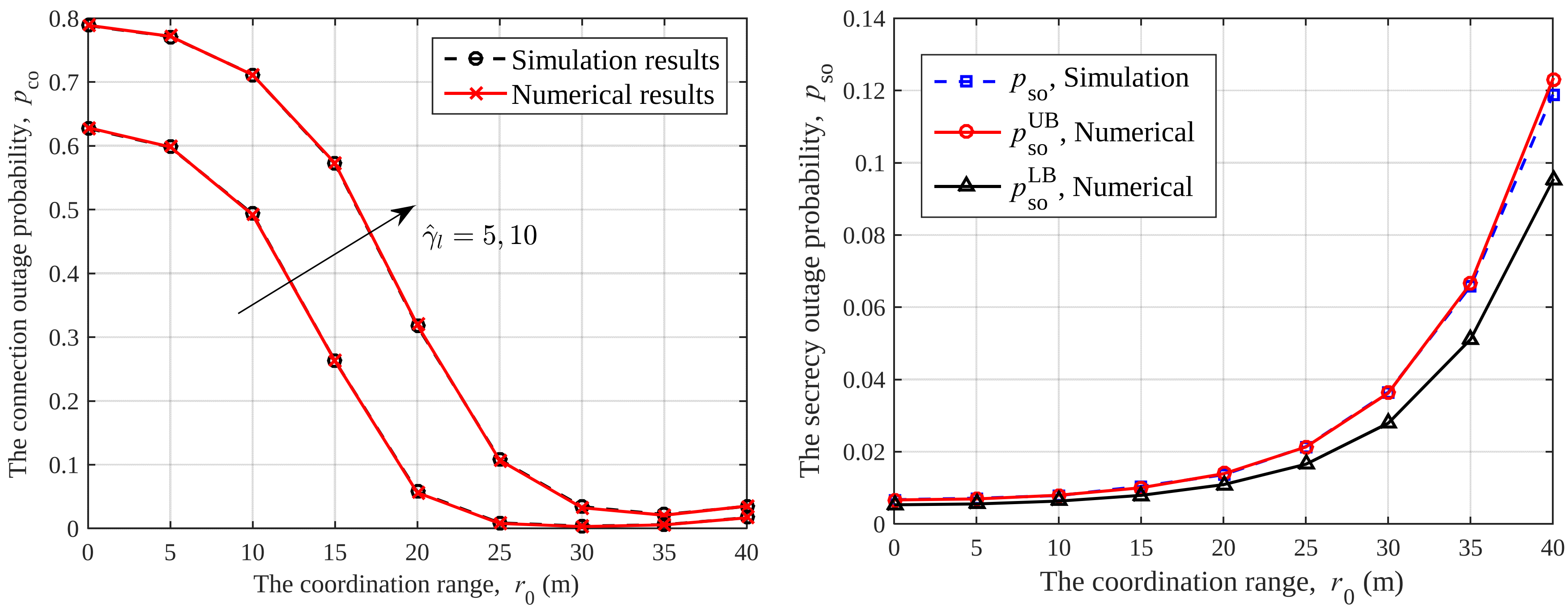
Connection vs. Secrecy Outage Trade-off



Increasing r_0 :

- mitigates more nearby interference to legitimate receivers;
- decreases the own link channel gain;
- decreases # of jamming streams.

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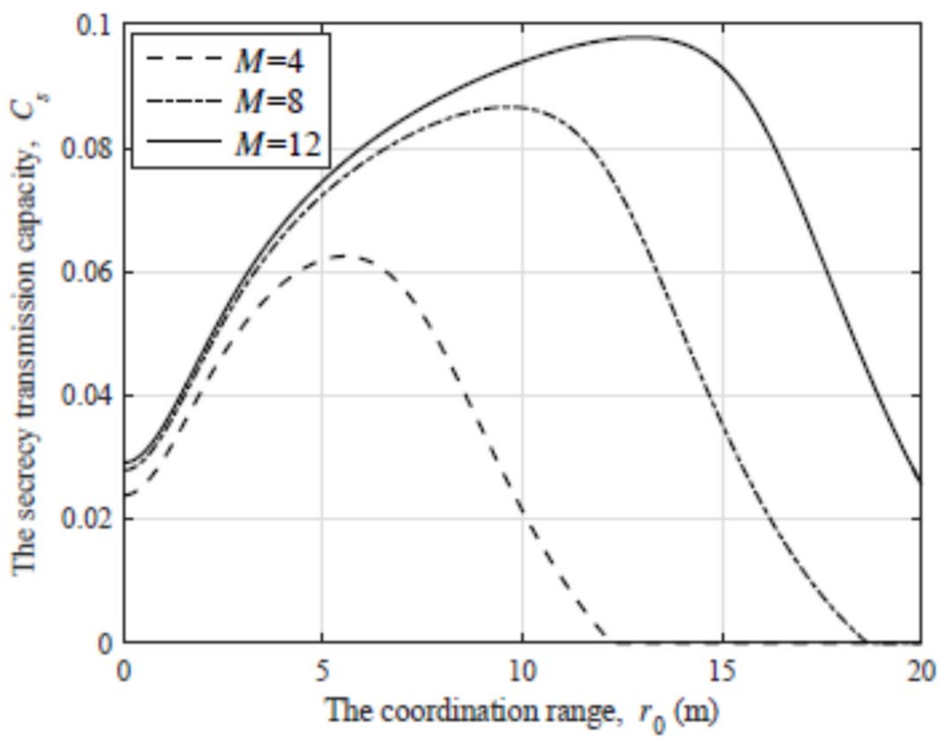
- Secrecy transmission capacity $C_s = (1 - p_{co})\lambda_l R_s$

  R_s: secrecy message rate

  - Achievable rate of confidential messages per unit area, with given connection and secrecy outage constraints



$C_s = \max_{r_0} C_s(r_0)$

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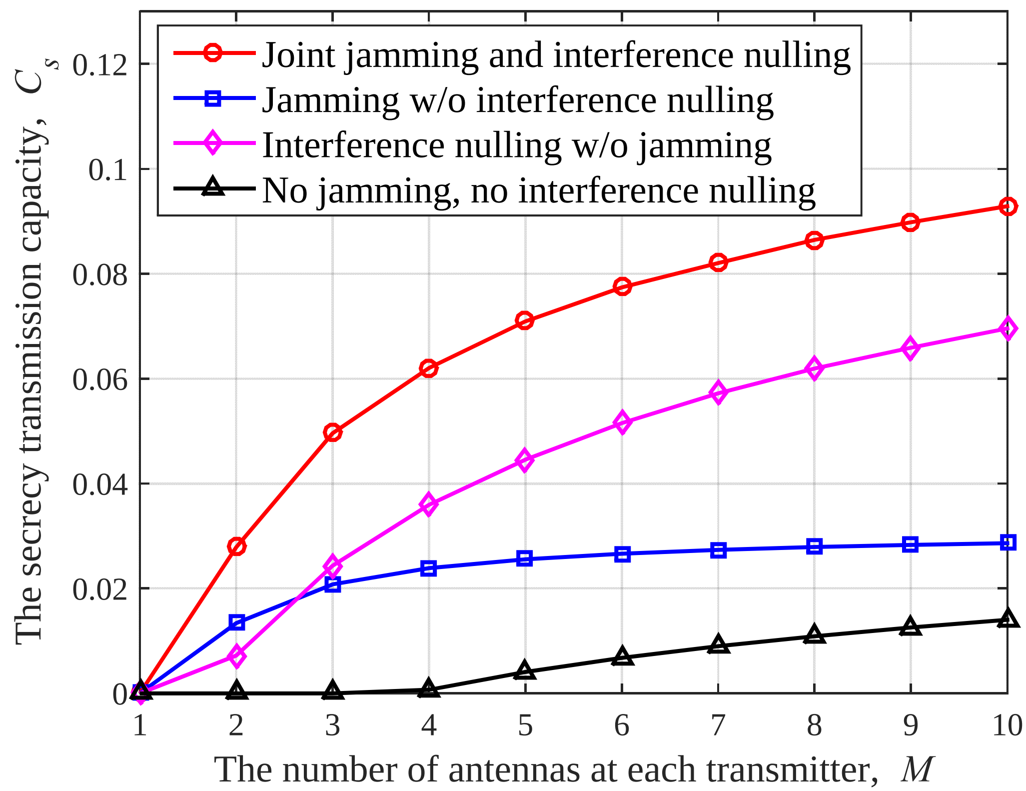
- Comparison of different approaches



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- Conclusions
  - A joint jamming and interference nulling scheme is proposed to enhance the network secrecy
  - The proposed scheme has large performance gain compared with the scheme only based on jamming, especially when the number of antennas is large
  - The results reveal the importance of interference nulling in jamming-aided networks

Conclusions

Conclusions

- A unified analytical framework for dense multi-antenna networks
  - Incorporates lots of existing analytical results on single- and multi-antenna networks as special cases
  - Almost as simple as the single-antenna case with Rayleigh fading
  - Abundant applications in dense networks
    - Coverage/outage, ASE, EE …
    - Small cells, HetNets, mm-wave …
    - ASE vs link reliability tradeoff
    - Interference coordination
    - PHY-layer security

Future Research Directions

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- Test the applicability of the proposed framework in more newly-emerging applications
  - UAV network
  - V2X network
  - …
- Further generalization, e.g., other more complicated distributions for the signal link, other spatial network models
  - May serve as good approximations for other models
  - *No model is perfect, simulations are needed to verify*

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Thanks

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